

New representations of the AES Key Schedule

Gaëtan Leurent, Clara Pernot
Inria, Paris

Friday, December 17th 2021

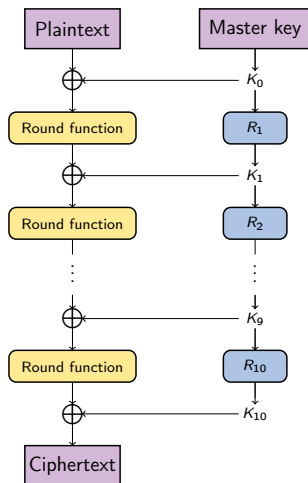
The Inria logo is written in a red, cursive script font.

AES: Advanced Encryption Standard [FIPS-197]

- The AES is the most widely used block cipher today.
- Winner of the AES competition.
- Subset of **Rijndael** block cipher.
- Designed by Rijmen and Daemen.
- **Block size:** 128 bits.
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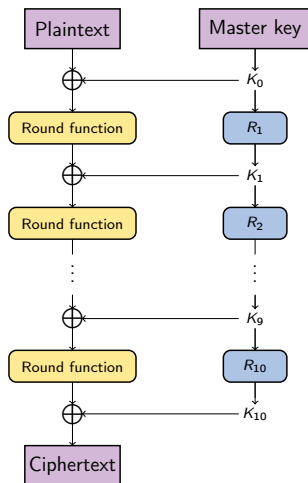
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- only **7 rounds out of 10** are broken.
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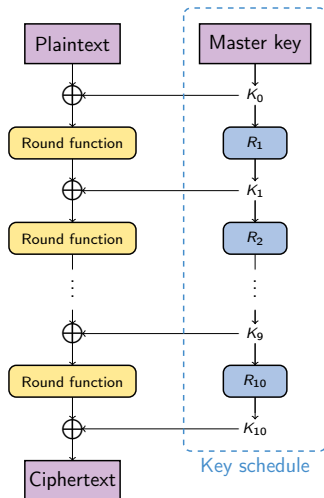
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AES Key Schedule

The AES key schedule is used to derive **11 subkeys** from a master key K .

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Division of the key into words and representation of the words in a matrix.

AES Key Schedule

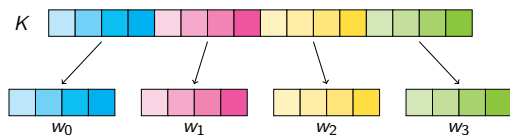
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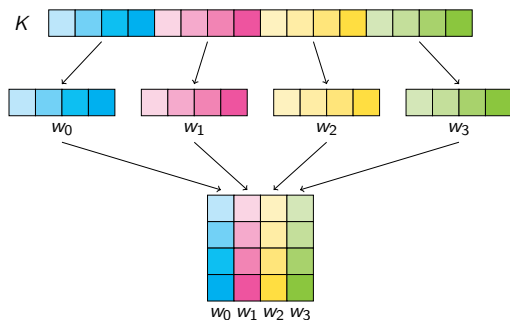
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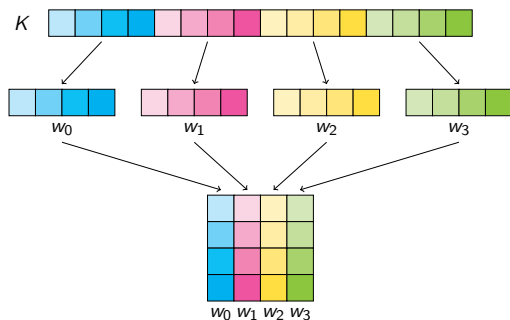
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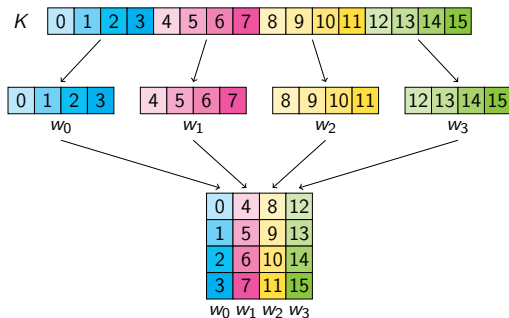


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→ The subkey at round i is the concatenation of the words w_{4i} to w_{3+4i} .

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AES Key Schedule

$$K_0 = K$$

| | w_0 | w_1 | w_2 | w_3 |
|--|-------|-------|-------|-------|
| | | | | |
| | | | | |
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Construction of words w_i for $i \geq 4$.

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| | | | | |

K_1

Construction of words w_i for $i \geq 4$.

AES Key Schedule

The leftmost column:

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|--|-------|-------|-------|-------|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

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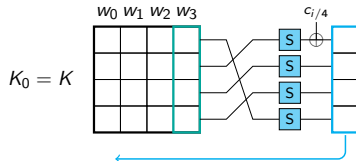
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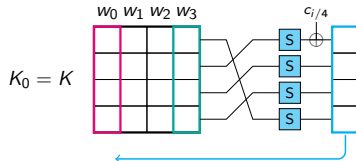
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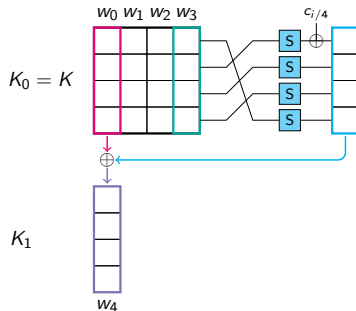
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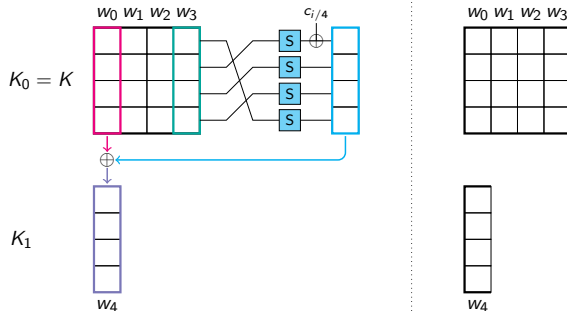


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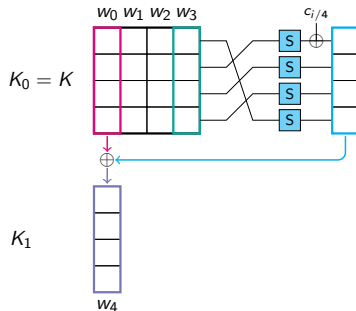


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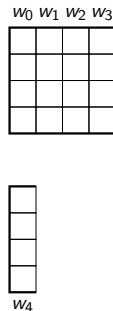
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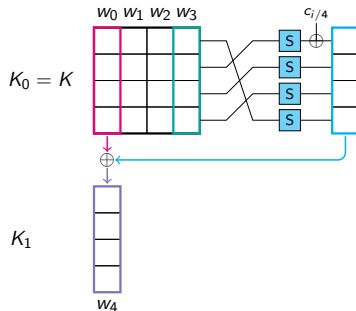


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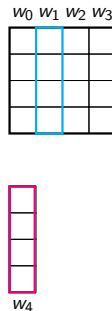
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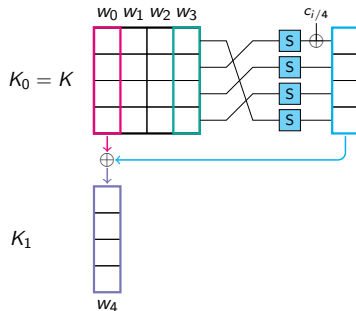


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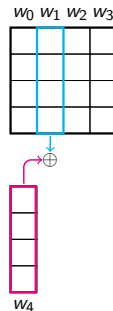
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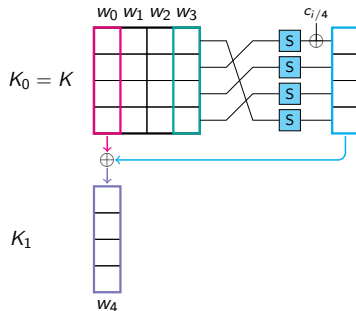


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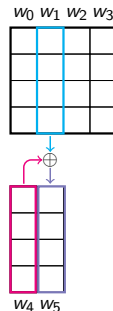
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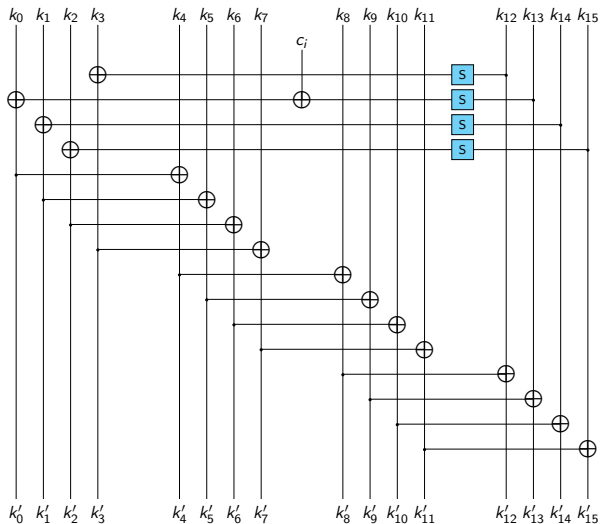
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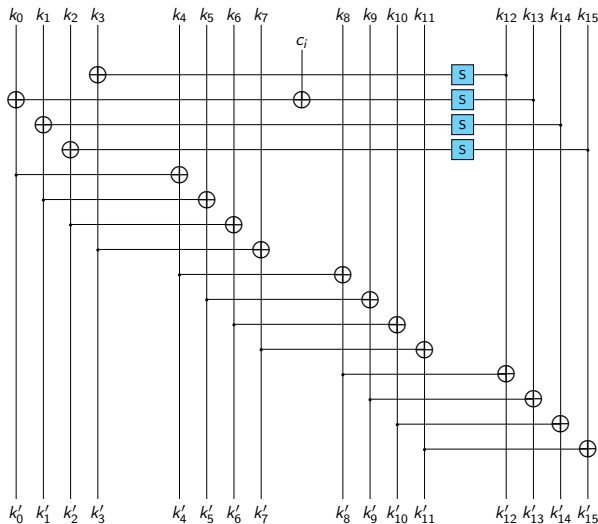
AES key schedule



One round of the AES key schedule.

AES key schedule

**Impression:
all bytes are mixed!**



One round of the AES key schedule.

Our results

- **Alternative representations** of the AES key schedules

**Even after a large number of rounds,
the key schedule does not mix all the bytes!**

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Even after a large number of rounds,
the key schedule does not mix all the bytes!

- **Short length cycles** when iterating an odd number of rounds of key schedule
 - ▶ Attacks on **mixFeed** and **ALE**
- **Efficient combination of information** from subkeys
 - ▶ Improvement of **Impossible Differential** and **Square** attacks against the AES

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Difference diffusion

Invariant subspaces: a subspace A and an offset u such as:

$$\exists u, F(A + u) = A + F(u)$$

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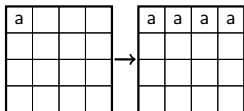
⇒ [LMR, EC'15] introduced an algorithm to **detect invariant subspaces**

Difference diffusion

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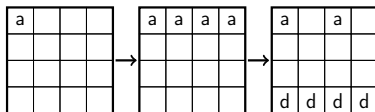
Diffusion of a difference on the first byte after several rounds of key schedule.

Difference diffusion



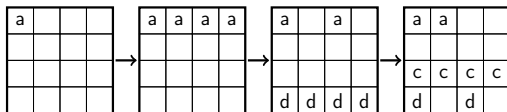
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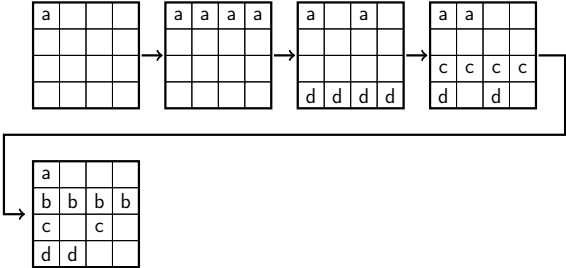
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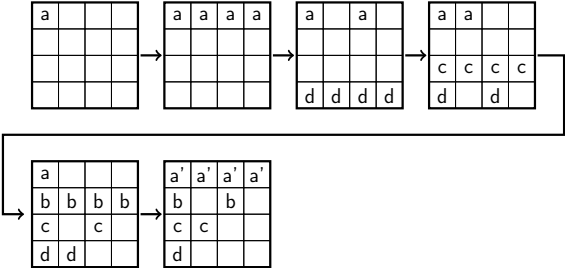
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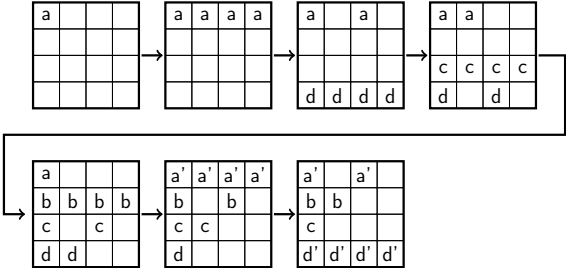
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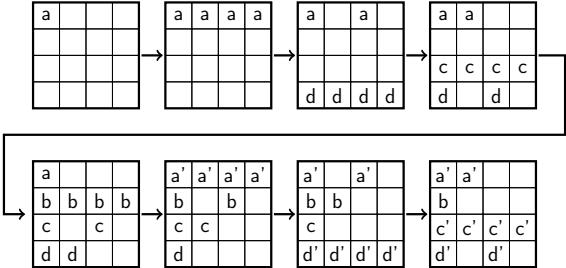
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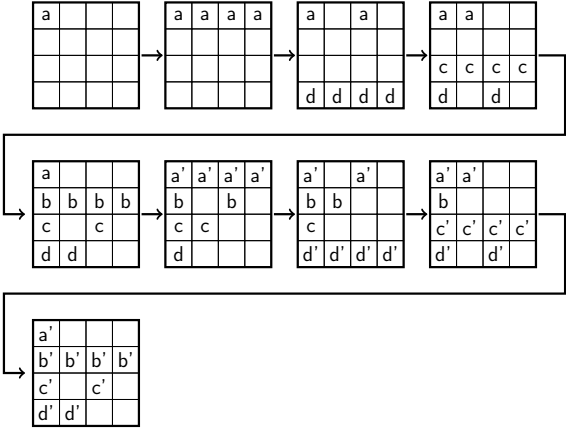
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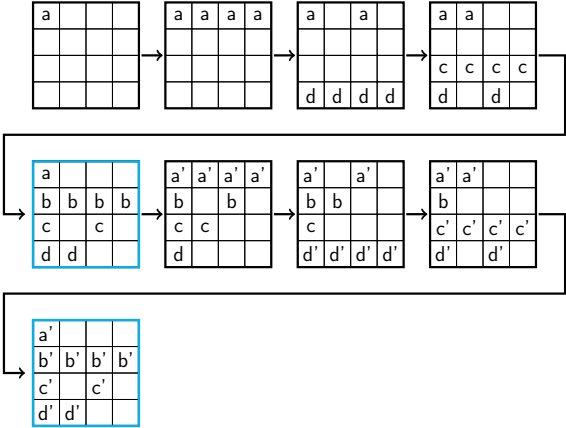
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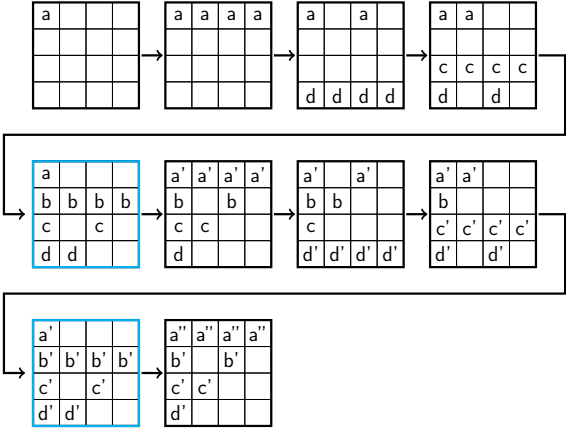
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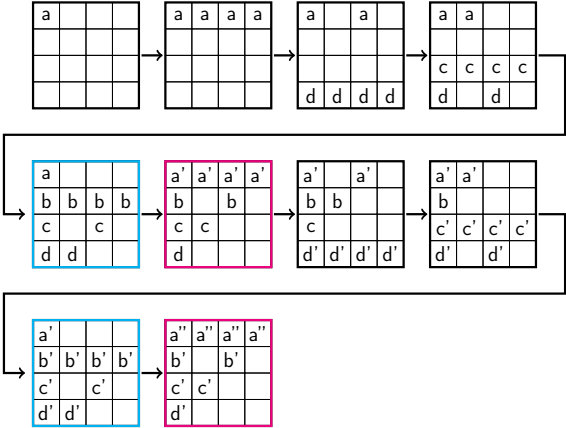
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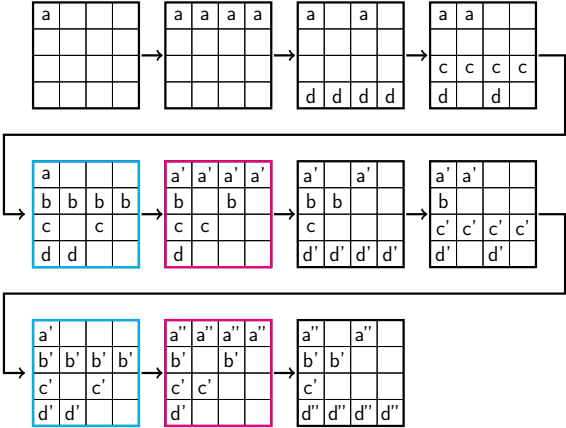
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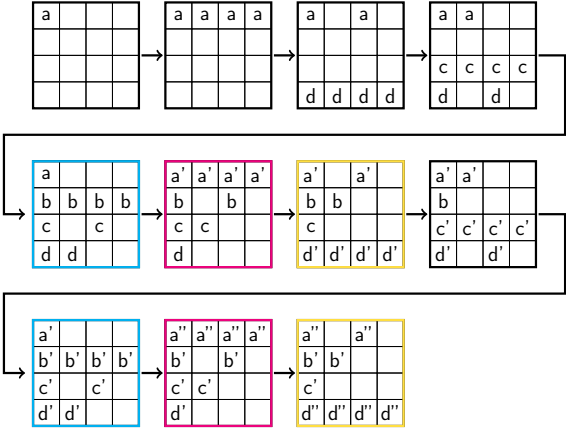
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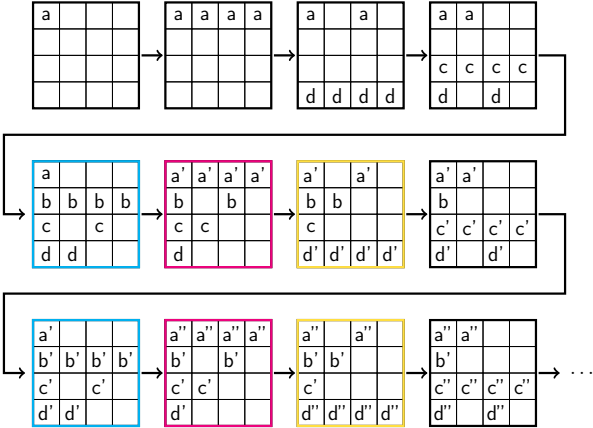
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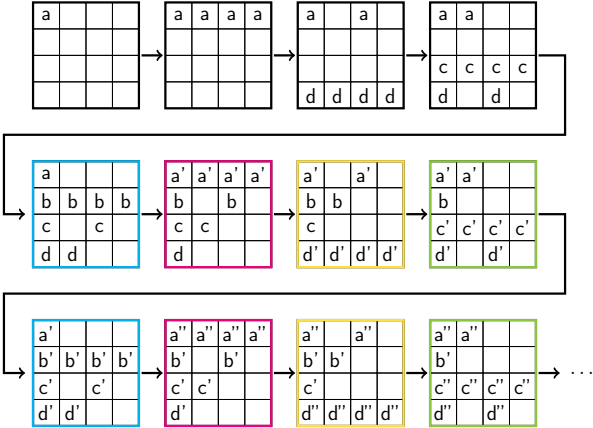
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Difference diffusion

We obtain **4 families of subspace trails** whose linear parts are:

- $E_0 = \{(a, b, c, d, 0, b, 0, d, a, 0, 0, d, 0, 0, 0, d) \text{ with } a, b, c, d \in \mathbb{F}_{2^8}\}$
- $E_1 = \{(a, b, c, d, a, 0, c, 0, 0, 0, c, d, 0, 0, c, 0) \text{ with } a, b, c, d \in \mathbb{F}_{2^8}\}$
- $E_2 = \{(a, b, c, d, 0, b, 0, d, 0, b, c, 0, 0, b, 0, 0) \text{ with } a, b, c, d \in \mathbb{F}_{2^8}\}$
- $E_3 = \{(a, b, c, d, a, 0, c, 0, a, b, 0, 0, a, 0, 0, 0) \text{ with } a, b, c, d \in \mathbb{F}_{2^8}\}$

$$\forall u \in (\mathbb{F}_{2^8})^{16}, R(E_i + u) = E_{i+1} + R(u)$$

The full space is the direct sum of those four vector spaces:

$$(\mathbb{F}_{2^8})^{16} = E_0 \oplus E_1 \oplus E_2 \oplus E_3$$

New representation of the AES Key Schedule

We perform a **linear transformation** $A = C_0^{-1}$, which corresponds to a change of basis:

Basis of E_0 :

$$s_0 = k_{15} \quad s_1 = k_{14} \oplus k_{10} \oplus k_6 \oplus k_2 \quad s_2 = k_{13} \oplus k_5 \quad s_3 = k_{12} \oplus k_8$$

Basis of E_1 :

$$s_4 = k_{14} \quad s_5 = k_{13} \oplus k_9 \oplus k_5 \oplus k_1 \quad s_6 = k_{12} \oplus k_4 \quad s_7 = k_{15} \oplus k_{11}$$

Basis of E_2 :

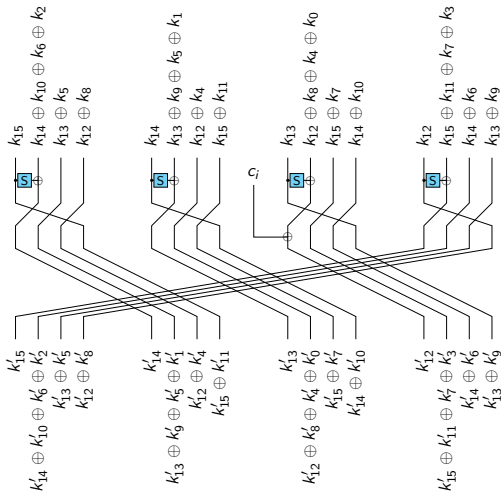
$$s_8 = k_{13} \quad s_9 = k_{12} \oplus k_8 \oplus k_4 \oplus k_0 \quad s_{10} = k_{15} \oplus k_7 \quad s_{11} = k_{14} \oplus k_{10}$$

Basis of E_3 :

$$s_{12} = k_{12} \quad s_{13} = k_{15} \oplus k_{11} \oplus k_7 \oplus k_3 \quad s_{14} = k_{14} \oplus k_6 \quad s_{15} = k_{13} \oplus k_9$$

\Rightarrow The **4 subspaces** appear more clearly!

New representation of the AES Key Schedule

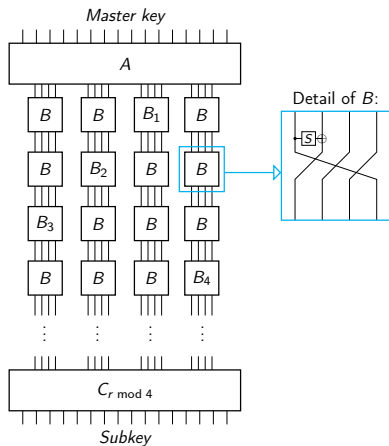


- 4 subspace trails
- 4 independent functions

The key schedule does not mix all the bytes!

One round of the AES key schedule
(alternative representation).

New representation of the AES Key Schedule



- B_i is similar to B but the round constant c_i is XORed to the output of the S-box.
- $C_i = A^{-1} \times SR^i$, with SR the matrix corresponding to rotation of 4 bytes to the right.

r rounds of the key schedule in the new representation.

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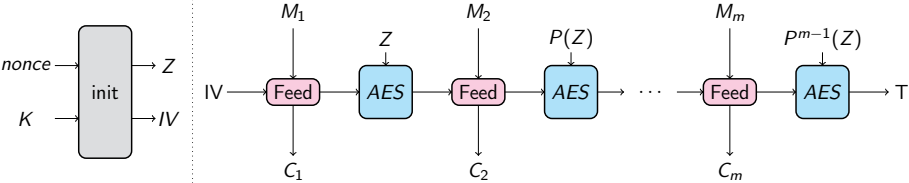
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mixFeed [Chakraborty and Nandi, NIST LW Submission]

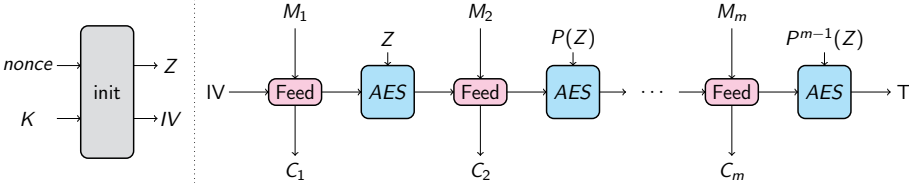
- mixFeed was a **second-round candidate** in the NIST Lightweight Standardization Process which was **not selected as a finalist**
- Submitted by Bishwajit **Chakraborty** and Mridul **Nandi**
- **AEAD** (Authenticated Encryption with Associated Data) algorithm
- Based on the AES block cipher

mixFeed

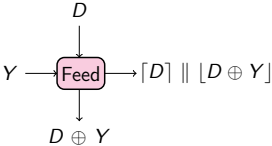


Simplified scheme of mixFeed encryption.

mixFeed

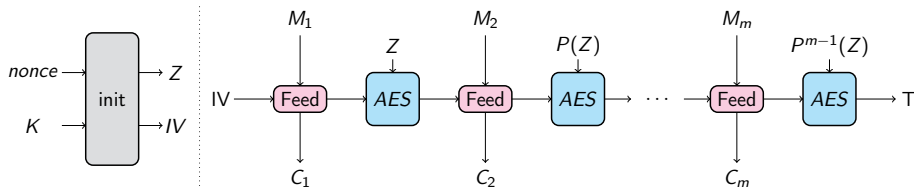


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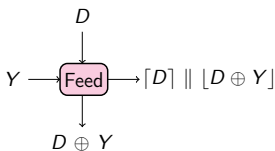


Function Feed in the case where $|D| = 128$

mixFeed



Simplified scheme of mixFeed encryption.



P : **11 rounds** of key schedule

P is **iterated** \rightarrow we study its **cycles!**

Function Feed in the case where
 $|D| = 128$

Mustafa Khairallah's observation [ToSC'19]

| |
|----------------------------------|
| 000102030405060708090a0b0c0d0e0f |
| 00020406080a0c0e10121416181a1c1e |
| 0004080c1014181c2024282c3034383c |
| 00081018202830384048505860687078 |
| 00102030405060708090a0b0c0d0e0f0 |
| 101112131415161718191a1b1c1d1e1f |
| 20222426282a2c2e30323436383a3c3e |
| 4044484c5054585c6064686c7074787c |
| 80889098a0a8b0b8c0c8d0d8e0e8f0f8 |
| 303132333435363738393a3b3c3d3e3f |
| 707172737475767778797a7b7c7d7e7f |
| 000306090c0f1215181b1e2124272a2d |
| 00050a0f14191e23282d32373c41464b |
| 00070e151c232a31383f464d545b6269 |
| 000d1a2734414e5b6875828f9ca9b6c3 |
| 00152a3f54697e93a8bdd2e7fc11263b |
| 00172e455c738aa1b8cfe6fd142b4259 |
| 00183048607890a8c0d8f00820385068 |
| 001c3854708ca8c4e0fc1834506c88a4 |
| 001f3e5d7c9bbad9f81736557493b2d1 |

Using brute-force and out of 33 tests,
Khairallah found **20 cycles of length**

$$14018661024 \approx 2^{33.7}$$

for the P permutation.

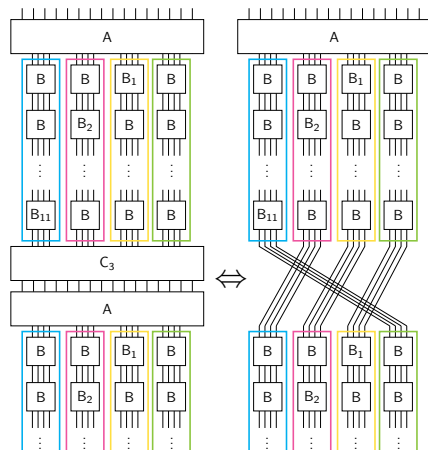
Surprising facts:

- all cycles found are of the **same length**
- this length is **much smaller** than the cycle length expected for a 128-bit permutation

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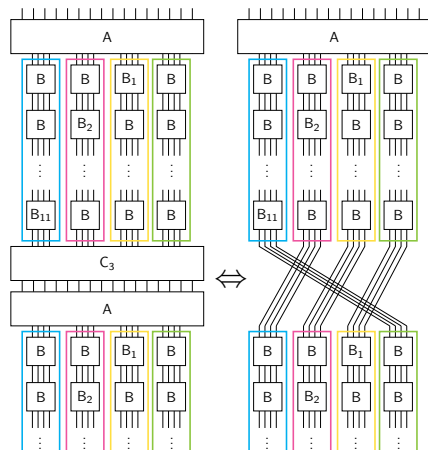
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Cycle analysis of 11-round AES key schedule



Two iterations of 11 rounds of the key schedule in the new representation.

Cycle analysis of 11-round AES key schedule



We define:

$$f_1 = B_{11} \circ B \circ B \circ B \circ B_7 \circ B \circ B \circ B \circ B_3 \circ B \circ B$$

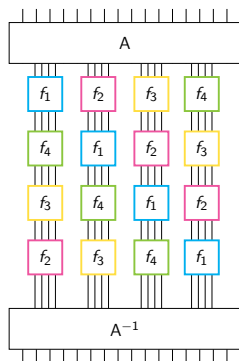
$$f_2 = B \circ B_{10} \circ B \circ B \circ B \circ B_6 \circ B \circ B \circ B \circ B_2 \circ B$$

$$f_3 = B \circ B \circ B_9 \circ B \circ B \circ B \circ B_5 \circ B \circ B \circ B \circ B_1$$

$$f_4 = B \circ B \circ B \circ B_8 \circ B \circ B \circ B \circ B_4 \circ B \circ B \circ B$$

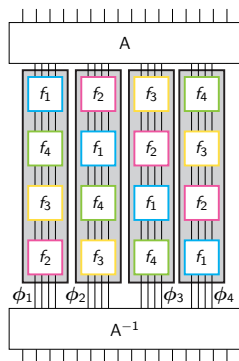
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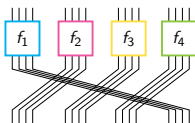


4 iterations of P in the new model.

Cycle analysis of 11-round AES key schedule



4 iterations of P in the new model.



$$\tilde{P} = A \circ P \circ A^{-1}$$

$$\tilde{P} : (a, b, c, d) \mapsto (f_2(b), f_3(c), f_4(d), f_1(a))$$

$$\tilde{P}^4 : (a, b, c, d) \mapsto (\phi_1(a), \phi_2(b), \phi_3(c), \phi_4(d))$$

$$\phi_1(a) = f_2 \circ f_3 \circ f_4 \circ f_1(a)$$

$$\phi_2(b) = f_3 \circ f_4 \circ f_1 \circ f_2(b)$$

$$\phi_3(c) = f_4 \circ f_1 \circ f_2 \circ f_3(c)$$

$$\phi_4(d) = f_1 \circ f_2 \circ f_3 \circ f_4(d)$$

Cycle analysis of 11-round AES key schedule

- If (a, b, c, d) is in a cycle of length ℓ of \tilde{P}^4 , we have:

$$\phi_1^\ell(a) = a \quad \phi_2^\ell(b) = b \quad \phi_3^\ell(c) = c \quad \phi_4^\ell(d) = d$$

In particular, a , b , c and d must be in cycles of ϕ_1 , ϕ_2 , ϕ_3 , ϕ_4 (respectively) of **length dividing ℓ** .

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- Conversely, if a, b, c, d are in cycles of the corresponding ϕ_i , then (a, b, c, d) is in a cycle of \tilde{P}^4 of length the **lowest common multiple of the small cycle lengths**.
- Due to the structure of the ϕ_i functions, all of them have the **same cycle structure**:

$$\phi_2 = f_2^{-1} \circ \phi_1 \circ f_2; \quad \phi_3 = f_3^{-1} \circ \phi_2 \circ f_3; \quad \phi_4 = f_4^{-1} \circ \phi_3 \circ f_4$$

Cycle analysis of 11-round AES key schedule

| Length | # cycles | Proba | Smallest element |
|------------|----------|----------------------|------------------|
| 3504665256 | 1 | 0.82 | 00 00 00 01 |
| 255703222 | 1 | 0.05 | 00 00 00 0b |
| 219107352 | 1 | 0.05 | 00 00 00 1d |
| 174977807 | 1 | 0.04 | 00 00 00 00 |
| 99678312 | 1 | 0.02 | 00 00 00 21 |
| 13792740 | 1 | 0.003 | 00 00 00 75 |
| 8820469 | 1 | $2^{-8,93}$ | 00 00 00 24 |
| 7619847 | 1 | $2^{-9,14}$ | 00 00 00 c1 |
| 5442633 | 1 | $2^{-9,63}$ | 00 00 02 78 |
| 4214934 | 1 | 2^{-10} | 00 00 05 77 |
| 459548 | 1 | $2^{-13,2}$ | 00 00 38 fe |
| 444656 | 1 | $2^{-13,24}$ | 00 00 0b 68 |
| 14977 | 1 | $2^{-18,13}$ | 00 06 82 5c |
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| 5165 | 1 | $2^{-19,67}$ | 00 0a d4 4e |
| 4347 | 1 | $2^{-19,92}$ | 00 04 94 3a |
| 1091 | 1 | $2^{-21,91}$ | 00 21 4b 3b |
| 317 | 1 | $2^{-23,7}$ | 00 28 41 36 |
| 27 | 1 | $2^{-27,25}$ | 01 3a 0d 0c |
| 6 | 1 | $2^{-29,42}$ | 06 23 25 51 |
| 5 | 3 | $3 \cdot 2^{-29,68}$ | 06 1a ea 18 |
| 4 | 2 | $2 \cdot 2^{-30}$ | 23 c6 f6 2b |
| 2 | 3 | $3 \cdot 2^{-31}$ | 69 ea 63 75 |
| 1 | 2 | $2 \cdot 2^{-32}$ | 7e be d1 92 |

Cycle structure of ϕ_1 for 11-round
AES-128 key schedule.

Cycle analysis of 11-round AES key schedule

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With probability $0.82^4 \simeq 0.45$, we have a , b , c and d in a cycle of length $\ell = 3504665256$, resulting in:

- a cycle of length ℓ for \tilde{P}^4 ,
- a cycle of length at most $4\ell = 14018661024$ for \tilde{P} and P .

Cycle structure of ϕ_1 for 11-round AES-128 key schedule.

Cycle analysis of 11-round AES key schedule

Summary: 45% of keys belong to cycles of length $14018661024 \approx 2^{33.7}$.

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Summary: 45% of keys belong to cycles of length $14018661024 \approx 2^{33.7}$.

- This explains the observation on mixFeed [Khairallah, ToSC'19].
- This allows to make a forgery against mixFeed.
- This contradicts the assumption made in a security proof of mixFeed:

Assumption [Chakraborty and Nandi, NIST LW Workshop]

For any $K \in \{0, 1\}^n$ chosen uniformly at random, probability that K has a period at most ℓ is at most $\ell/2^{n/2}$.

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Forgery attack against mixFeed [Khairallah, ToSC'19]

The goal of a **forgery attack** is to forge a valid tag T' for a new ciphertext C' using (M, C, T) .

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- we assume that Z belongs to a **cycle** of length ℓ
- we choose a message M made of m blocks, with $m > \ell$

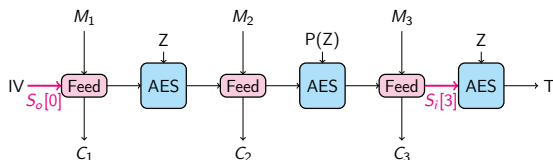
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(1) Cut



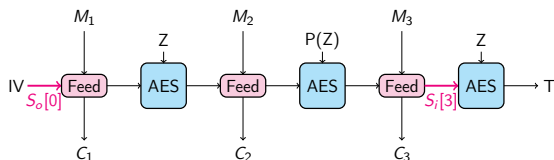
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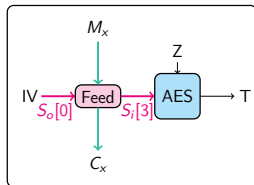
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(1) Cut



(2) Paste



Forgery attack against mixFeed

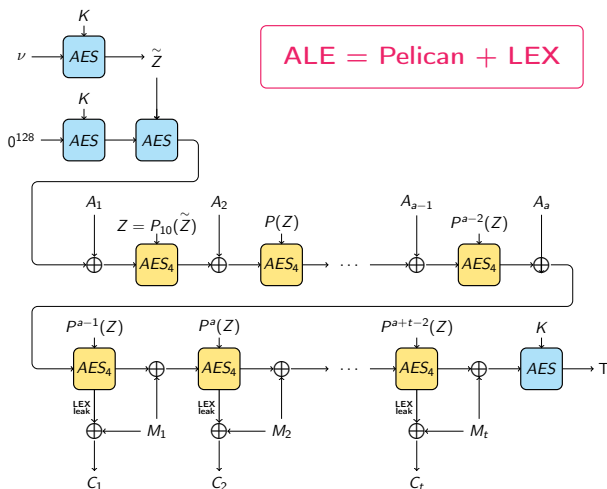
Summary of the forgery attack:

- Data complexity: a known plaintext of length higher than $2^{37.7}$ bytes
 - Memory complexity: negligible
 - Time complexity: negligible
 - Success rate: 45%
- ⇒ Verified using the mixFeed reference implementation:
41 successes out of 100 tests!

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Application to ALE [FSE:BMRR13]



Authenticated encryption with ALE.

→ Each AES encryption is expected to be performed with different keys.

Application to ALE

- Using the same approach as for mixFeed: 76% of the keys belong to cycles of length $16043203220 \approx 2^{33.9}$.
- Short length cycles allows to find states encrypted under the same key.
- New attacks are found using [\[BDF, Crypto'11\]](#).

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| Attack | | Enc | Verif | Time | Ref |
|--------------------------------------|------------------|-------------|-----------|-------------|------------------------------|
| Existential Forgery | Known Plaintext | $2^{110.4}$ | 2^{102} | $2^{110.4}$ | [Wu+, AC'13] |
| Existential Forgery | Known Plaintext | 2^{103} | 2^{103} | 2^{104} | [KR, SAC'14] |
| Existential Forgery | Known Plaintext | 1 | 2^{120} | 2^{120} | [KR, SAC'14] |
| State Recovery, Almost Univ. Forgery | Known Plaintext | 1 | 2^{121} | 2^{121} | [KR, SAC'14] |
| State Recovery, Almost Univ. Forgery | Chosen Plaintext | $2^{57.3}$ | 0 | $2^{104.4}$ | New |

Comparison of attacks against ALE.

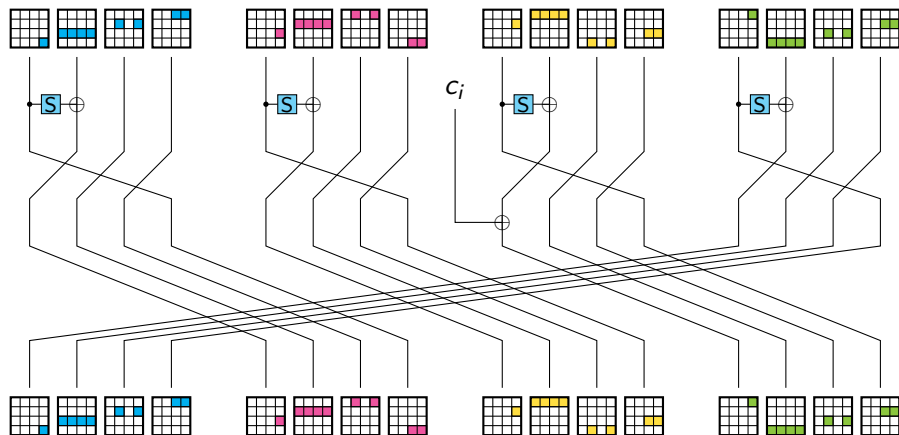
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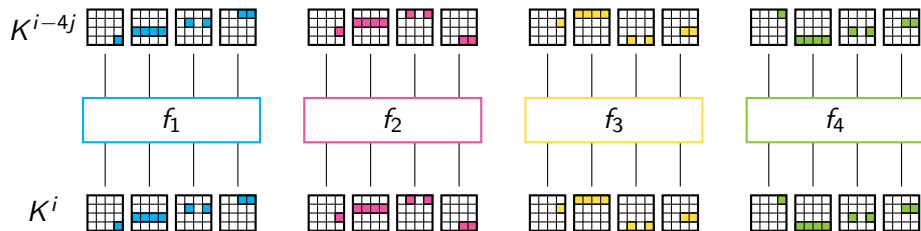
Property on the AES Key Schedule



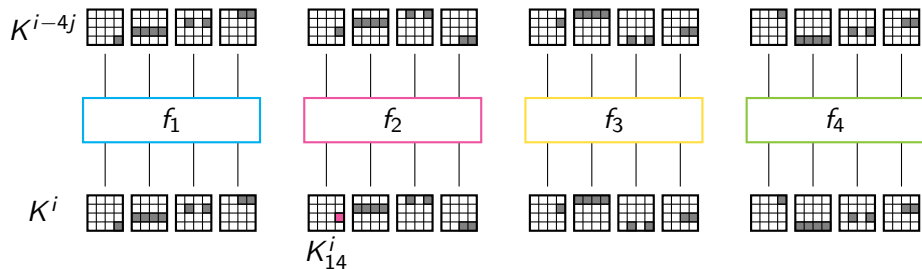
One round of the AES key schedule with graphic representations of bytes positions (alternative representation).

Only the XOR of the colored bytes is required for each state.

Property on the AES Key Schedule

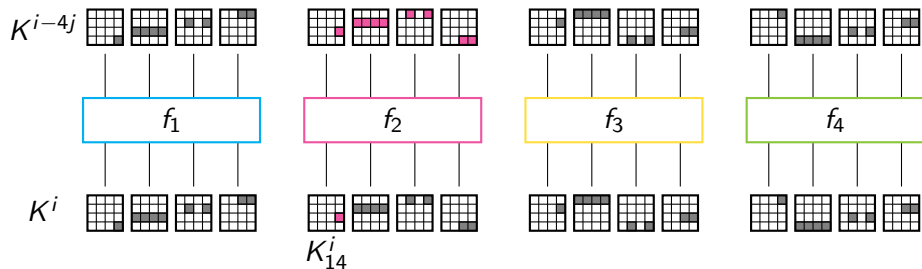


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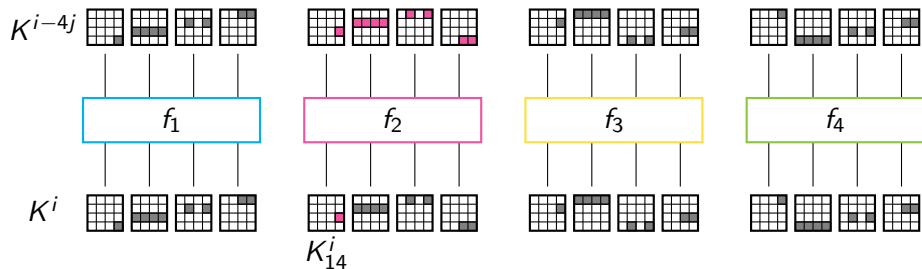
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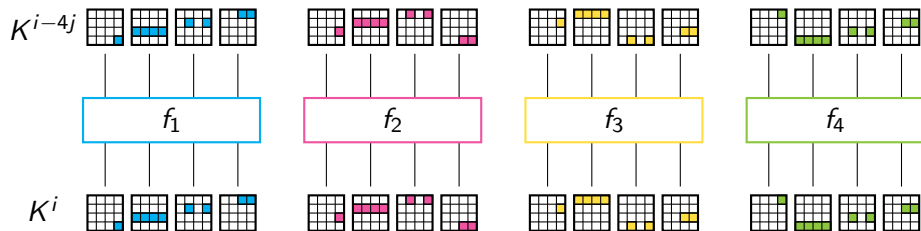
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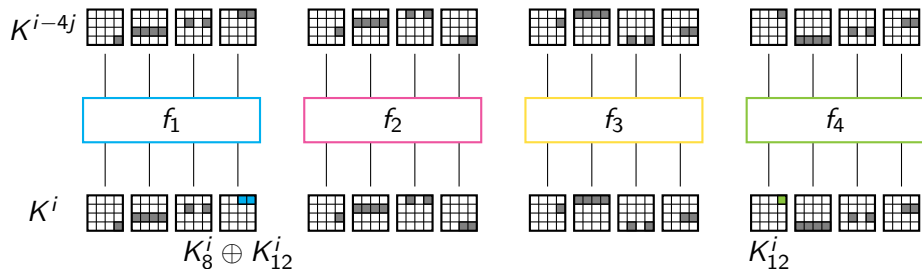
→ A byte in the last column depends on only 32 bits of information.

Property on the AES Key Schedule



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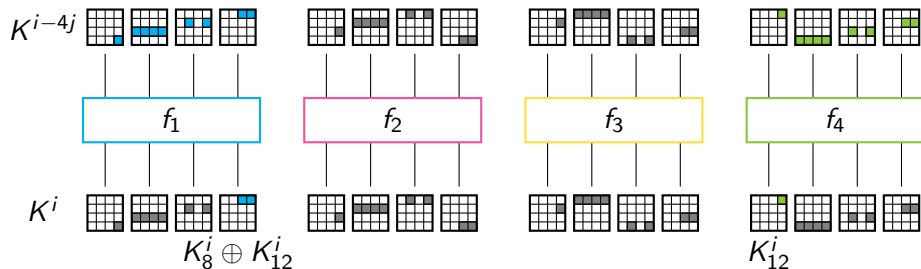


How to compute K_8^i ?

$$K_8^i = (K_8^i \oplus K_{12}^i) \oplus K_{12}^i$$

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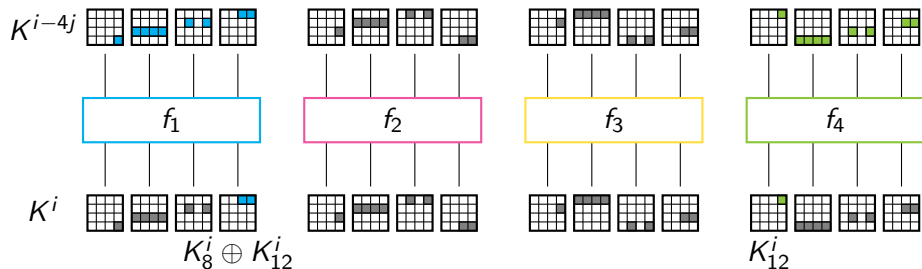


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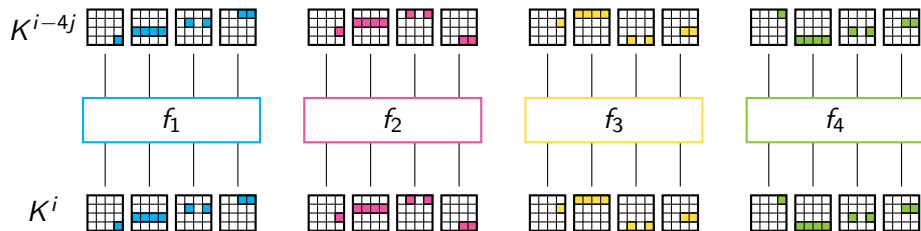


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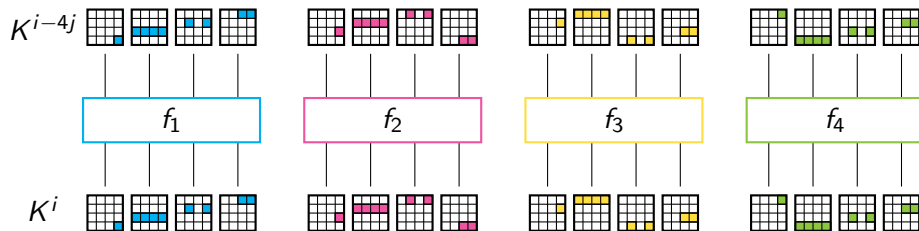
- A byte in the last column depends on only 32 bits of information.
- **A byte in the 3rd column depends on only 64 bits of information.**

Property on the AES Key Schedule



- A byte in the last column depends on only 32 bits of information.
- A byte in the 3rd column depends on only 64 bits of information.
- **A byte in the 2nd column depends on only 64 bits of information.**

Property on the AES Key Schedule



- A byte in the last column depends on only 32 bits of information.
- A byte in the 3rd column depends on only 64 bits of information.
- A byte in the 2nd column depends on only 64 bits of information.
- **A byte in the first column depends on 128 bits of information.**

Property on the AES Key Schedule

Using our **new representation** of the key schedule, we demonstrate that:

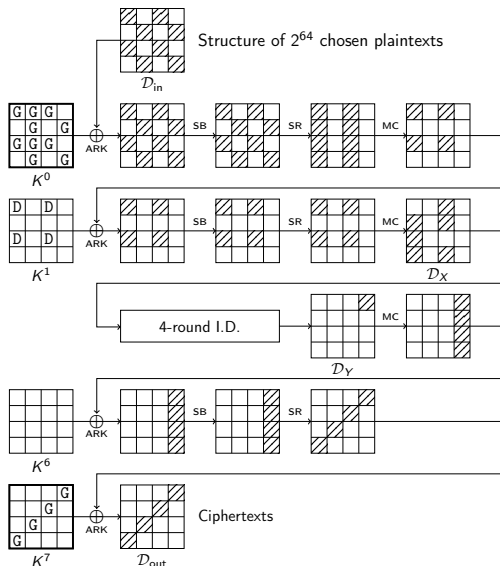
- A byte in the **last** column depends on only **32 bits** of information
- A byte in the **3rd** column depends on only **64 bits** of information
- A byte in the **2nd** column depends on only **64 bits** of information
- A byte in the **first** column depends on **128 bits** of information

**Even after a large number of rounds,
the key schedule does not mix all the bytes!**

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Impossible Differential – AES



The attack is in 2 parts:

- (1) find candidates for the key bytes marked G.
- (2) find the master keys corresponding to these bytes.

7-round impossible differential attack ([MDRM, IC'10]).
Figure adapted from Tizk for Cryptographers [Jean].

Matching bytes from K^0 and K^7

Given 10 bytes of K^0 and 4 bytes of K^7 ,
how to find the corresponding master keys?

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- Guess 6 bytes of K^0
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Complexity: 2^{48}

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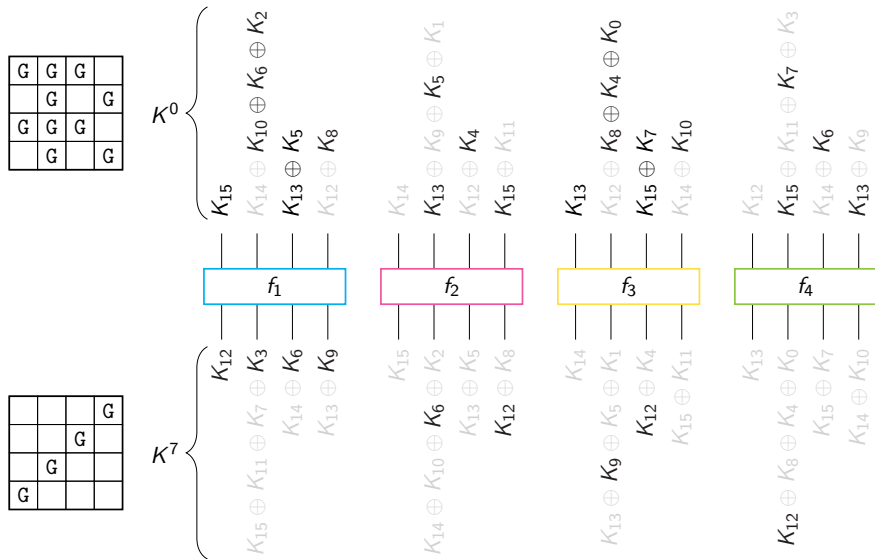
Improvement:

- Guess 2 bytes of K^0
- Filter using 2 bytes of K^7
- Guess 2 bytes of K^0
- Filter using 1 byte of K^7
- Guess 1 byte of K^0
- Deduce 1 byte of K^0 from K^7

Complexity: 2^{48}

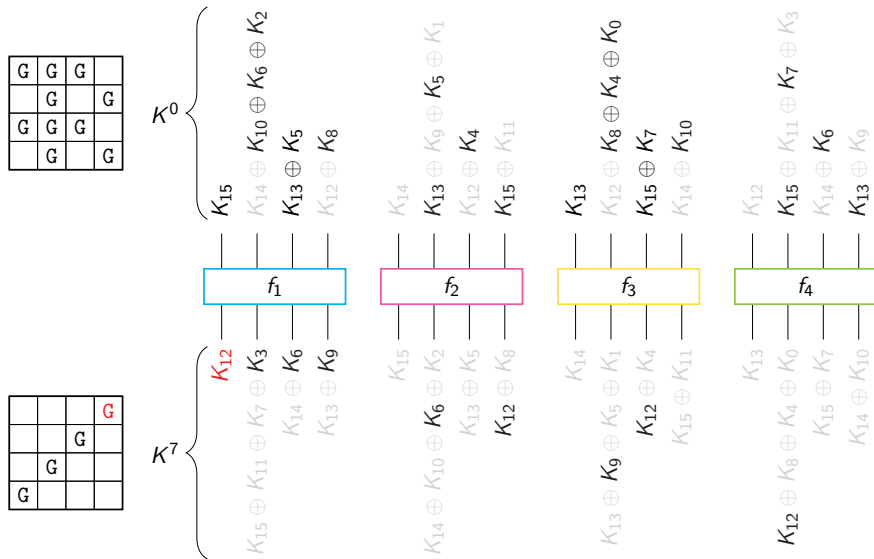
Complexity: 4×2^{16}

Matching bytes from K^0 and K^7



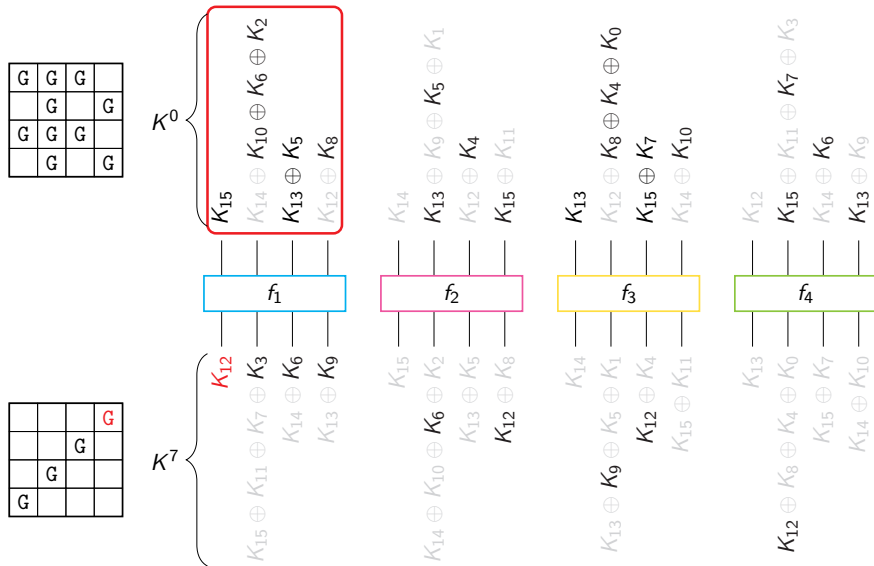
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How to compute K_{12}^7 from K^0 ?



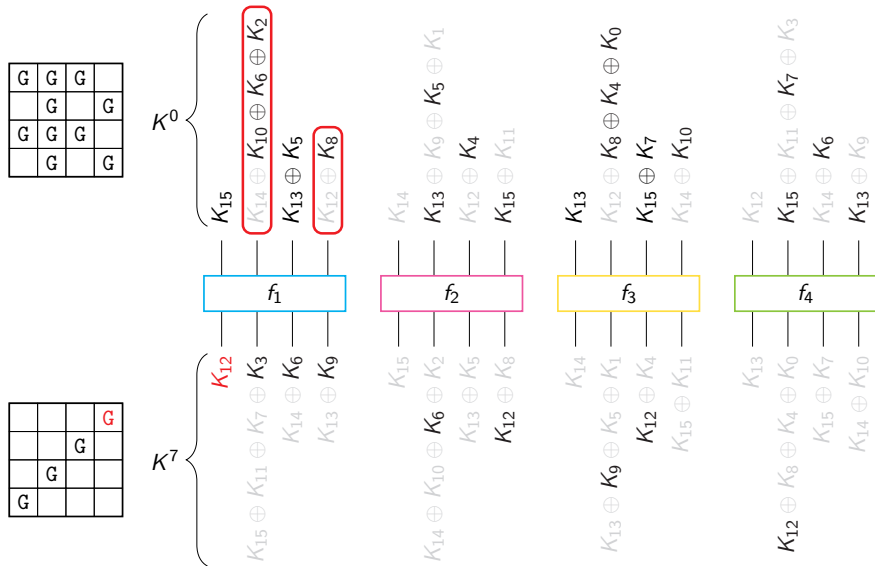
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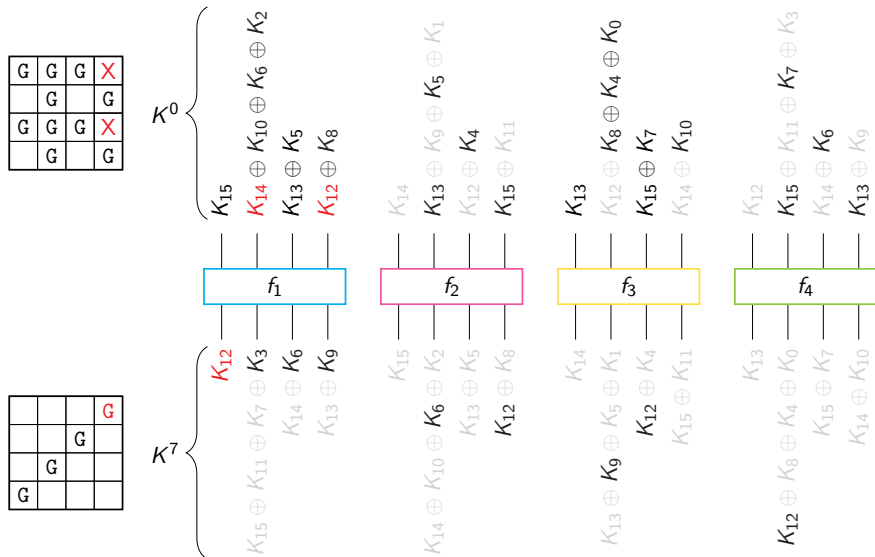
Matching bytes from K^0 and K^7

How to compute K_{12}^7 from K^0 ?

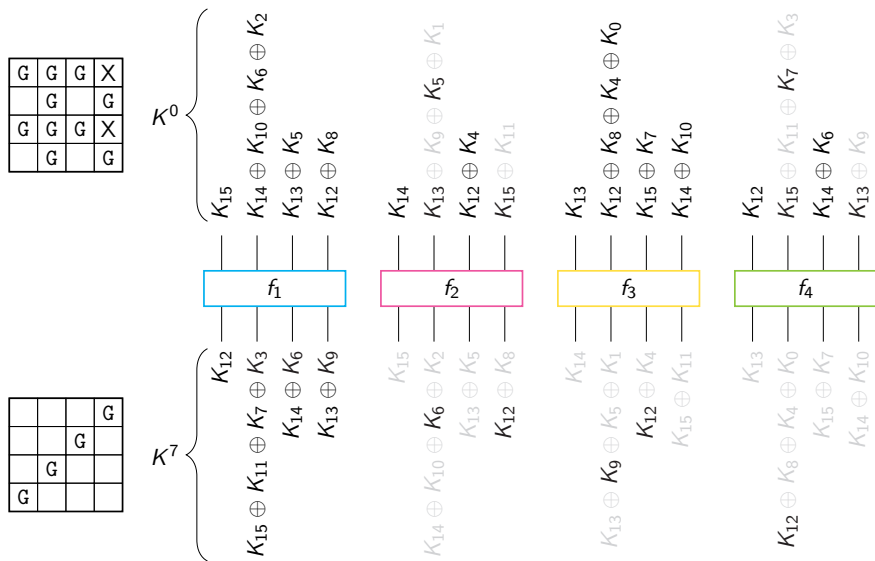


Matching bytes from K^0 and K^7

We can filter using K_{12}^7 by guessing only 2 bytes of K^0 !

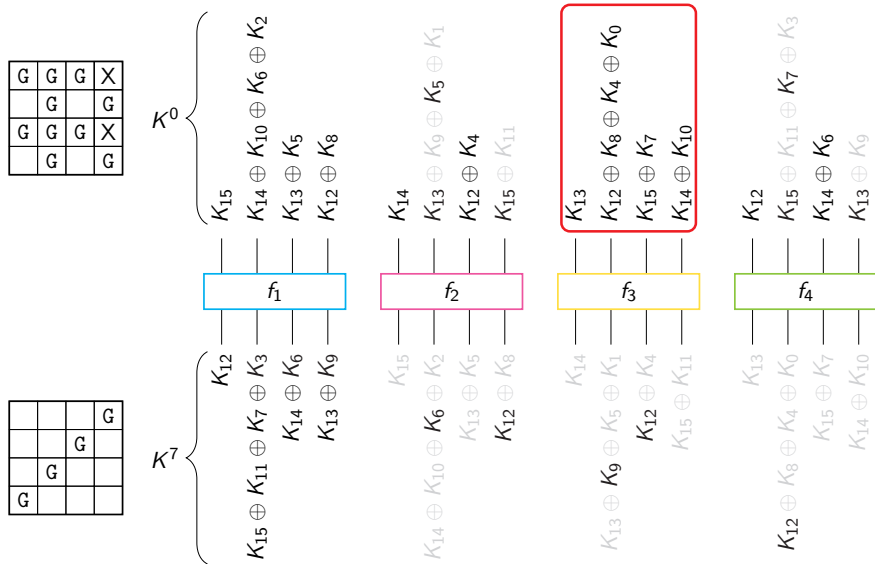


Matching bytes from K^0 and K^7



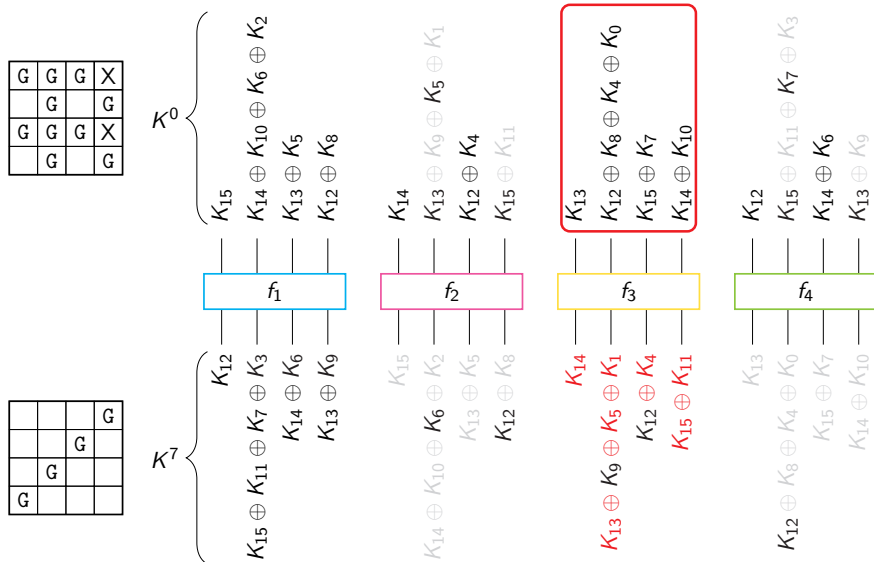
Matching bytes from K^0 and K^7

All the input of f_3 is known, so the output is also known

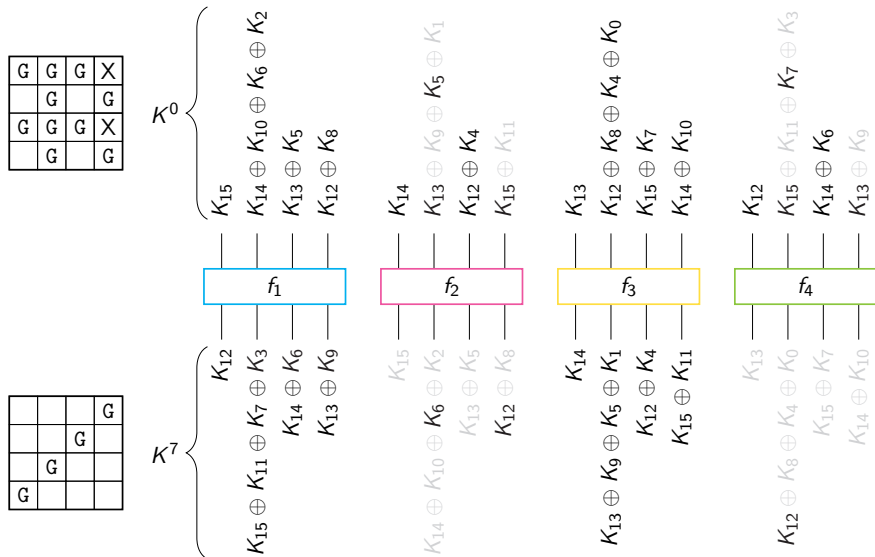


Matching bytes from K^0 and K^7

All the input of f_3 is known, so the output is also known

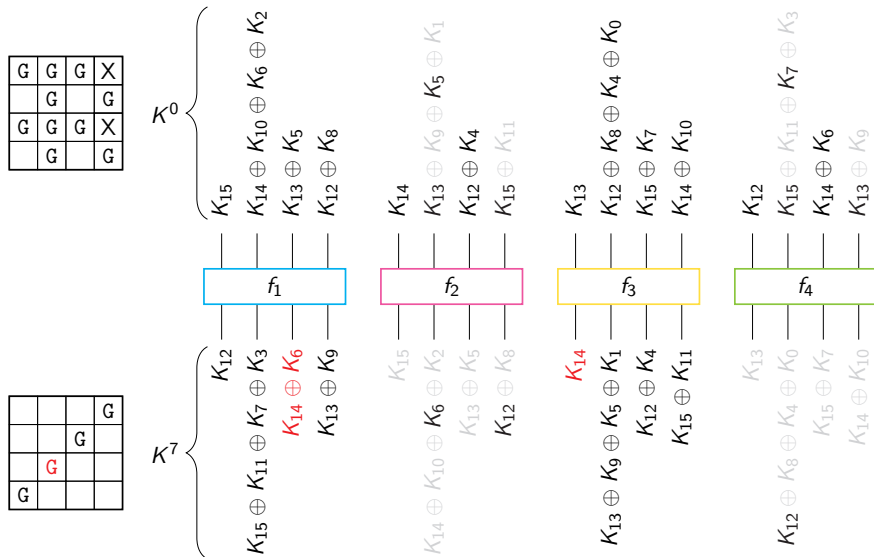


Matching bytes from K^0 and K^7



Matching bytes from K^0 and K^7

We are also able to filter according to $K_6^7 = (K_{14}^7 \oplus K_6^7) \oplus K_{14}^7$



Results

| Attack | Data | Time | Mem. | Ref. |
|-------------------------|-------------|-------------|------------|---|
| Meet-in-the-middle | 2^{97} | 2^{99} | 2^{98} | [Derbez, Fouque, Jean, EC'13] |
| | 2^{105} | 2^{105} | 2^{90} | [Derbez, Fouque, Jean, EC'13] |
| | 2^{105} | 2^{105} | 2^{81} | [Bonnetain, Naya-Plasencia, Schrottenloher, ToSC'19] |
| | 2^{113} | 2^{113} | 2^{74} | [Bonnetain, Naya-Plasencia, Schrottenloher, ToSC'19] |
| Impossible differential | 2^{113} | 2^{113} | 2^{74} | [Boura, Lallemand, Naya-Plasencia, Suder, JC'18] |
| | $2^{105.1}$ | 2^{113} | $2^{74.1}$ | [Boura, Lallemand, Naya-Plasencia, Suder, JC'18] |
| | $2^{106.1}$ | $2^{112.1}$ | $2^{73.1}$ | Variant of [Boura, Lallemand, Naya-Plasencia, Suder, JC'18] |
| | $2^{104.9}$ | $2^{110.9}$ | $2^{71.9}$ | New |

Best single-key attacks against 7-round AES-128.

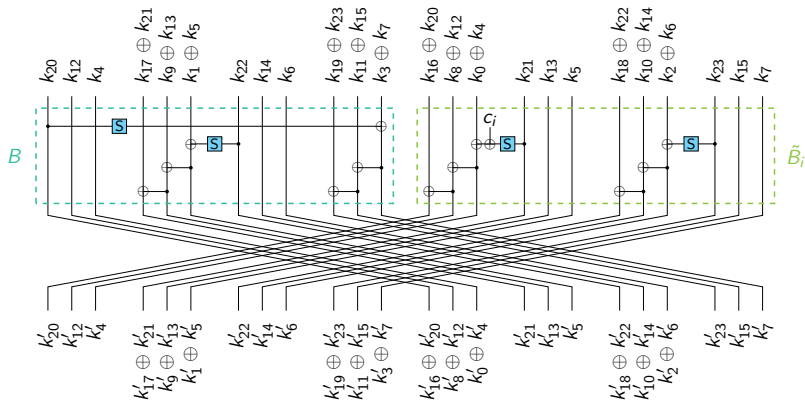
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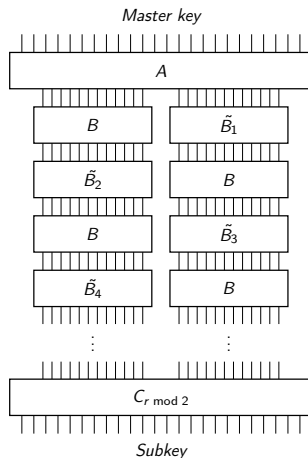
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New Representation of the AES-192 Key Schedules



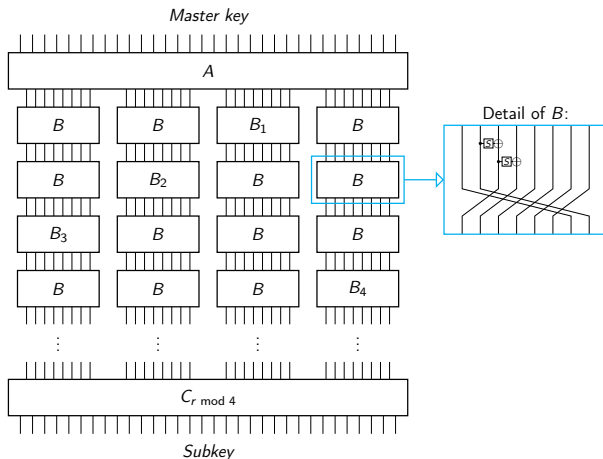
One round of the AES-192 key schedule (alternative representation).

New Representation of the AES-192 Key Schedules



r rounds of the AES-192 key schedule in the new representation.

New Representation of the AES-256 Key Schedules



r rounds of the AES-256 key schedule in the new representation. B_i is similar to B but the round constant c_i is XORed to the output of the first S-box.

Other Results

| Attack | Cipher | Rounds | Data | Time | Reference |
|-------------------------------------|------------------|--------|---------------------|-------------|-----------------------------|
| Square | AES-192 | 8/12 | $2^{128} - 2^{119}$ | 2^{188} | [FKL+, FSE'00] |
| | | | $2^{128} - 2^{119}$ | $2^{187.3}$ | Variant of [FKL+, FSE'00] |
| | | | $2^{128} - 2^{119}$ | $2^{185.7}$ | Variant of [DKS, AC'10] |
| | | | $2^{128} - 2^{119}$ | $2^{185.1}$ | New |
| Related-Key Impossible Differential | AES-192 | 8/12 | $2^{64.5}$ | 2^{177} | [ZWZ+, SAC'06] |
| | | | $2^{63.5}$ | 2^{175} | New |
| Impossible Differential | Rijndael-256/256 | 9/14 | $2^{229.3}$ | 2^{194} | [WGR+, ICISC'12] |
| | | | $2^{228.1}$ | $2^{192.9}$ | Variant of [WGR+, ICISC'12] |
| | | | $2^{227.6}$ | $2^{192.5}$ | New |
| Impossible Differential | Rijndael-256/256 | 10/14 | $2^{244.2}$ | $2^{253.9}$ | [WGR+, ICISC'12] |
| | | | $2^{243.9}$ | $2^{253.6}$ | Variant of [WGR+, ICISC'12] |
| | | | 2^{242} | $2^{251.7}$ | New |

Square Attack

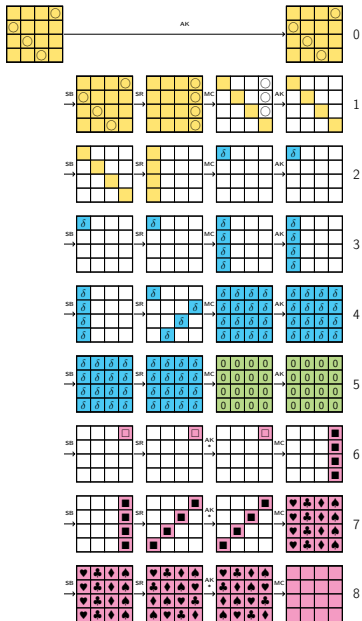
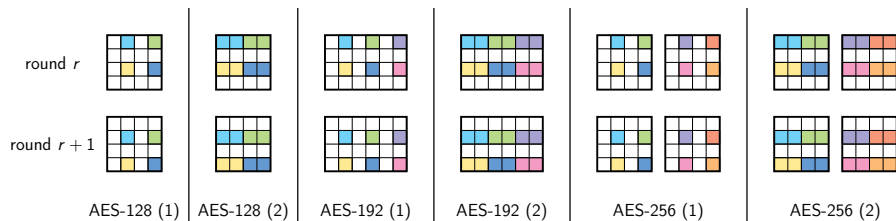


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Properties on the AES Key Schedule



Representation of the position of the bytes of the proposition.

In cases (2), only the XOR of the two bytes of the same color must be known.

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Conclusion

→ **Alternatives representations** of AES key schedules:

- ▶ 128 bits: 4 chunks of 4 bytes
- ▶ 192 bits: 2 chunks of 12 bytes
- ▶ 256 bits: 4 chunks of 8 bytes

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For more details:

<https://eprint.iacr.org/2020/1253>