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Calibration of parallel robots : on the Elimination of Pose–Dependent Parameters

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We motivate and study the use of the forward kinematic model in the calibration scheme of parallel robots. Classical methods uses this model based only on the internal sensors for parameters elimination. One aim of the paper is to demonstrate how, in this well-constrained case, the model has numerous different solutions, how selecting a solution can lead to critical difficulties and possibly erroneous results. We propose two kind of alternatives, one drawing upon the methods of computer algebra to explicit the relation between the set of sensors and the parameters, and another which minimize the residual errors of an over-constrained formulation of the forward kinematics. This propositions are substantiated by experiments with planar robots, which are easy to describe completely and offer a full understanding of the underlying modeling and behavior.

1 Introduction

1.1 Motivation

The kinematics calibration of closed-loop robots has been studied in the last decade. Its formulation is well described, namely by Wampler in Wampler et al. (1995).

Kinematic equations of such a robot are non-linear equations $F(\mathbf{p}, \mathbf{q}, \mathbf{x}) = 0$ relating the kinematics parameters $\mathbf{p}$ which are independent on the poses, the active joint variables $\mathbf{q}$ whose values may be obtained by measurements, and unknown parameters $\mathbf{x}$, which are dependent on the poses of the robot (passive joint variables, generalized coordinates, ...).

Due to assembly and fabrication errors, the actual values of $\mathbf{p}$ are not precisely known. In some applications, for instance in the cases of cable robots used for recovery in inaccessible areas on the sites of natural disasters, even a rough approximation of the values of $\mathbf{p}$ are not known Tadokoro et al. (1999). Hence, a thorough study of the kinematics calibration and of its sensibility to the quality of the initial approximation is interesting.

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1.2 Calibration of Parallel Robots

For the determination of the kinematics parameters $\mathbf{p}$, it is necessary to write as least as many equations as the number of parameters $\mathbf{p}$, added to the total number of pose–dependent unknown parameters $\mathbf{x}$.

For any pose k of the robot, the corresponding instance of the kinematics inverse model $F_k(\mathbf{p}, \mathbf{q}_k, \mathbf{x}_k)$ provides equations involving the kinematics parameters $\mathbf{p}$, and also the unknown parameters $\mathbf{x}_k$ corresponding to that pose. Hence, multiplying the number of poses to get more equations is only interesting if the number of equations obtained for one pose is at least greater than the number of the corresponding parameters $\mathbf{x}_k$.

For that purpose external measurement devices Zhuang et al. (1998), redundant internal sensor Zhuang (1997); Chiu and Perng (2002); Rauf et al. (2004) and/or extra mechanical constraints Huang et al. (2003) are widely used. Then, a sufficient number n of poses are considered, depending on the numerical method used for the identification of the parameters $\mathbf{p}$. Resulting is a well or over–constrained system

$$\mathcal{F}(\mathbf{p}, \mathbf{q}_1, \ldots, \mathbf{q}_n, \mathbf{x}_1, \ldots, \mathbf{x}_n) = 0 \tag{1}$$

with an usually very large number of equations and unknown variables $\mathbf{x}$.

We have to estimate or eliminate these variables $\mathbf{x}$ so as to allow identification of the parameters $\mathbf{p}$.

Classical methods consist of using a numerical scheme for getting an estimation of $\mathbf{x}_k$ based on an initial guess $\mathbf{x}_k^0$, an approximation of $\mathbf{p}$, and a measurement of $\mathbf{q}_k$ – see Rauf et al. (2004).

Wampler insists in a footnote of Wampler et al. (1995) on the necessary quality of the initial guess $\mathbf{x}_k^0$: *"If some unmeasured joint displacement cannot be eliminated, they can be solved numerically as part of an iterative solution. However, the convergence of the method will then be dependent on having sufficiently good initial guesses for these joints"*. We will also consider the issue of the quality of the approximation of $\mathbf{p}$ and of the measurement of $\mathbf{q}_k$

The other main step of the parallel robot calibration consists in the identification process of the parameters $\mathbf{p}$. Once the $\mathbf{x}$ variables are eliminated, the equations are rewritten in

$$\mathcal{S}(\mathbf{p}, \mathbf{q}_1, \ldots, \mathbf{q}_n) = 0 \tag{2}$$

This system is classically over–constrained to allow to take into account the noise associated to the measurements.

In this case the approach consists in defining a cost function $C(\mathbf{p}, \mathbf{q})$ and to minimize this criterion with the help of a specific least-squares algorithm (Levenberg-Marquardt, conjugate gradient, quasi Newton). Usual definition for the cost function is the square of the norm of the residual error on the equations:

$$C(\mathbf{p}, \mathbf{q}) = \|\mathcal{S}(\mathbf{p}, \mathbf{q})\|^2 \tag{3}$$

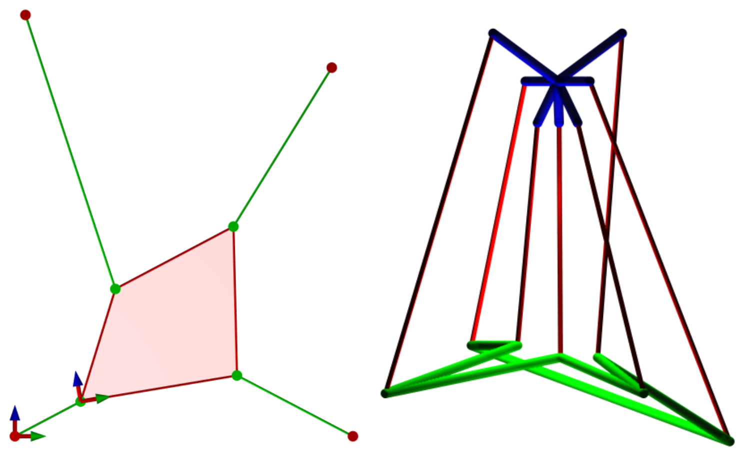
In this paper, we will focus on the elimination of the pose–dependent parameters, which is one of the less addressed topic in the existing literature, and which is, in our opinion, the intrinsic difference between calibration of serial robots and of parallel robots.

In the next section of this paper, we describe the example of the planar parallel robot, that we will study. The kinematics calibration problem of this robot is both one of the simplest instance of this problem and an already interesting one for the purpose of comparing methods and validating results. In the third section, we discuss methods for calibration and for elimination of the pose-dependent parameters and we propose several enhancements. In the last one, we present results of an experiment of simulation of calibration of the planar parallel robot in which we are comparing these methods.

2 Calibration of a 4-RPR Planar Robot

2.1 Presentation

We consider a 3-RPR planar robot with an extra leg (see Figure 1(a)), denoted as a 4-RPR planar robot.



(a) planar 4RPS (b) Gough 7RRPRRR

Figure 1: Parallel robots with an additional leg.

We consider n different configurations of this robot and, for each of them, the lengths of the four legs are measured by four internal sensors.

From this information, the self-calibration process aims to determine the coordinates of the attachment points of the legs to the mobile platform and to the base platform.

2.2 Motivation

We selected this robot for our study for the following reasons:

- The methods developed for this robot are easily extendible to the self-calibration of wire robots, which have become very popular recently, due to the wide potential of their applications – Tadokoro et al. (1999).
- The calibration problem is a 2-dimensional problem equivalent to the self-calibration of a Gough platform with one redundant leg 1(b), that models several kinds of calibration procedure (where the additional information is provided by a ball bar Takeda et al. (2002), by a single theodolite, or by measuring the norm of the position of the end-effector Daney and Emiris (2001)).
- In this case, it is possible to symbolically eliminate the pose-dependent parameters $\mathbf{p}$ involved in the inverse kinematics model, by applying algebraic methods on these equations.

2.3 Robot Description

The planar robot 4-RPR is made of two platforms: the base and the mobile platform. One reference frame, $\Omega_O = (O, x, y)$ (resp. $\Omega_C = (C, u, v)$) is attached to the base (resp. to the mobile platform). The two platforms are connected by four legs, each made of a revolute joint, a prismatic joint, and again a revolute joint. Attachment points of the legs to the base (resp. to the mobile platform) are denoted by A_i (resp. by B_i). For $i = 1, \ldots, 4$, we denote by a_i (resp. b_i) the vector OA_i (resp. CB_i) expressed in Ω_O (resp. in Ω_C). Position of the frames w.r.t. the platforms are given as follows:

$$x_{a_1} = y_{a_1} = y_{a_2} = x_{b_1} = y_{b_1} = y_{b_2} = 0 \tag{4}$$

2.4 Inverse Kinematics Model and Calibration Equations

The *inverse kinematics* model is used to obtain the calibration equation as:

$$\mathcal{IK}_{k,i} : \qquad \|P_k + R_k.b_i - a_i\|^2 \quad = \quad l_{k,i}^2 \tag{5}$$

where P_k (resp. R_k) is the relative position (resp. the orientation matrix) of Ω_C w.r.t. Ω_O for the k–th configuration of the robot and $l_{k,i}$ denotes the length of the i–th leg for the same configuration. We denote by J^{inv} the inverse kinematics Jacobian.

2.5 Forward Kinematics

The forward kinematics model of a planar 3-RPR is studied in Gosselin et al. (1992). Here, we extend this model for the over-constrained planar robot 4-RPR.

The aim is to determine the position $P = [x, y]$ and the orientation matrix $R(\theta)$ as a function of the kinematics parameters $\mathbf{p}_i = [a_i, b_i]$ and of the lengths l_i.

Assuming (4), $\mathcal{IK}_1$ may be rewritten in $x^2 + y^2 = l_1^2$. $\mathcal{IK}_2$, $\mathcal{IK}_3$, and $\mathcal{IK}_4$ become, by substitution of $x^2 + y^2$ by l_1^2, linear in x, y. It is then easy to get a symbolic expression for x and y using $\mathcal{IK}_2$ and $\mathcal{IK}_3$ and we substitute the result in $\mathcal{IK}_1$ and in $\mathcal{IK}_4$. We obtain, after simplification, two univariate polynomials (in $\tan \frac{\theta}{2}$) : U_3 of degree 6 and V of degree 4, whose coefficients are functions of the lengths and of the kinematics parameters.

If we exchange the equations corresponding to legs 3 and 4 in the above process, we can obtain another polynomial of degree 6, i.e. U_4.

The roots of U_3 (resp. U_4) give all the possible orientations of the robot if we do not consider the 4–th leg (resp. the 3–rd leg). The common roots of U_3 or U_4 and V give the orientation of the over-constrained planar robot 4–RPR. Thus we can determine x and y by solving the resulting system of linear equations.

It is well known that the forward kinematics model of the planar 3–RPR robot has at most 6 real solutions. In the case of 4–RPR, Husty in Husty M. (2001) shows an example (and the procedure to obtain it) such that the number of real solutions is also 6. This is a very important result because it shows that it is not possible to reduce the number of solutions to one for a redundant robot. He has also shown the same type of result for a Gough platform with one additional leg. If we apply the above symbolic resolution to this kind of robot, U_3 and U_4 are the same polynomial and the coefficients of V are all equal to zero.

In the next section, we will also consider two other types of resolution of the forward kinematics model:

- the 3–RPR *numerical* forward kinematics resolution uses an iterative scheme (like a Powell method) to converge to a pose solution, starting from an initial point. Note that the direction of convergence of the algorithm is chosen by the inverse kinematics Jacobian J^{inv}. In the case where we consider the 4–RPR, we can compute two 3–RPR *numerical* forward kinematics resolution for the two 3–RPR robots obtained by omitting the 3–rd or the 4–th leg. Due to the local convergence of this iterative scheme and its sensibility to the choice of an initial estimation, it is not possible to certify the convergence of both 3–RPR resolution to the same real position/orientation. The remark holds even if the robot is perfectly known or if the lengths of the legs are measured without noise.

- the 4–RPR *least-squares forward kinematics* uses an iterative scheme (like a Levenberg-Marquardt, a conjugate gradient, a quasi-Newton process, ...) to converge to a real approximation of the pose of the robot which minimizes the least-square error for the lengths of the legs. The scheme needs an initial point, and the direction of convergence is provided by the rectangular Jacobian J^{inv} extended for the 4 legs.

In the rest of this paper, we will refer to the former three methods respectively by $\mathcal{FK}_{symb}$, $\mathcal{FK}_{num}$ and $\mathcal{FK}_{lsq}$.

3 Definition of the Calibration Criterion

As described in the previous section, the inverse kinematics model provides $4n$ equations for n configurations of the robot. The system of equations $\mathcal{F}(\mathbf{p}, \mathbf{q}_1, \ldots, \mathbf{q}_n, \mathbf{x}_1, \ldots, \mathbf{x}_n) = 0$ is as follows:

$$
\begin{array}{ccccc}
\underbrace{\begin{array}{c}
\|P_1\|^2 - l_{1,1}^2 = 0 \\
\|P_1 + R_1.b_2 - a_2\|^2 - l_{1,2}^2 = 0 \\
\|P_1 + R_1.b_3 - a_3\|^2 - l_{1,3}^2 = 0 \\
\|P_1 + R_1.b_4 - a_4\|^2 - l_{1,4}^2 = 0
\end{array}}_{F_1(\mathbf{p}, \mathbf{x}_1) = 0}
& \ldots &
\underbrace{\begin{array}{c}
\|P_k\|^2 - l_{k,1}^2 = 0 \\
\|P_k + R_k.b_2 - a_2\|^2 - l_{k,2}^2 = 0 \\
\|P_k + R_k.b_3 - a_3\|^2 - l_{k,3}^2 = 0 \\
\|P_k + R_k.b_4 - a_4\|^2 - l_{k,4}^2 = 0
\end{array}}_{F_k(\mathbf{p}, \mathbf{x}_k) = 0}
& \ldots &
\underbrace{\begin{array}{c}
\|P_n\|^2 - l_{n,1}^2 = 0 \\
\|P_n + R_n.b_2 - a_2\|^2 - l_{n,2}^2 = 0 \\
\|P_n + R_n.b_3 - a_3\|^2 - l_{n,3}^2 = 0 \\
\|P_n + R_n.b_4 - a_4\|^2 - l_{n,4}^2 = 0
\end{array}}_{F_n(\mathbf{p}, \mathbf{x}_n) = 0}
\end{array}
$$

In this section, we will describe and compare the three main different approaches for defining the calibration criterion.

3.1 Basic Approach

A very simple and basic approach, – but up to our knowledge never used – consist in using as a calibration criterion the square of the norm of the sum of the equations $\|\mathcal{F}(\mathbf{p}, \mathbf{q}_1, \ldots, \mathbf{q}_n, \mathbf{x}_1, \ldots, \mathbf{x}_n)\|^2$ and to use a least-square minimization method for identification.

The unknowns that we have to identify are, in this case the kinematics parameters a_{ik}, b_{ik} and also the position/orientation of the measurement configurations.

The main drawback of this approach is the large number of unknowns, which is asymptotically proportional to the number of measurement poses n.

3.2 Error on additional Measurements

This approach is proposed by Patel in Patel and Ehmann (2000) for a Gough platform with one additional linear sensor. We adapt it below for the 3-RPR planar robot.

The method is based on the *inverse calibration* method of a parallel robot. It consists in minimizing the difference between the values provided by the additional sensor and the corresponding length calculated with an inverse kinematics model of the robot. As the generalized coordinates (the position and the orientation) are unknown, they are estimated by the forward kinematics as a function of the values provided by extra internal sensors.

In this case, the 4–th leg plays the role of the additional redundant sensor.

We derived the calibration equations by substituting, for each configuration k, the pose-dependent parameters P, R by the result $\bar{P}_k, \bar{R}_k$ of a forward kinematics resolution applied to the first three legs of the planar robot, into the equation corresponding to the 4–th leg:

$$S_k(\mathbf{p}, \mathbf{q}_k) = \left\|\bar{P}_k + \bar{R}_k.b_4 - a_4\right\|^2 - l_{k,4}^2 \tag{6}$$

A classical cost function based on residual error (see equation 3) is computed from these equations and used in a minimization process involving the kinematics parameters.

The key problem of this approach is the use of the forward kinematics resolution. For parallel robots, the solution of the forward kinematics model are, for each parameter, a set of many, real or complex values and it is important to distinguish between the solutions and to be able to *follow* a solution during the main iterative process of calibration, to insure the smoothness of the cost function and hence the validity of the minimization process.

- A closed-form of the solution is almost never possible to obtain except in very simple cases. The selection of one in the set of solutions is classically made using a criterion based on the distance between the solution and an initial estimation. When no initial estimation is available, we propose to use a criterion based on the residual error of the calibration equations. It is important to note that complex solutions can not be neglected because, due to measurement and estimation errors of kinematic parameters, closed-forms only produce approximations of the actual solution.

- In the other cases, a $\mathcal{FK}_{num}$ process is used to determine an approximation of one real solution. This process needs an initial estimation of the pose-dependent parameters, but even if this estimation is closed to the corresponding values for the actual pose, there is no guarantee at all for the process to converge to that pose. As a matter of fact, we do not have any guarantee in this case on the smoothness of the cost function, and hence on the convergence of the minimization algorithm.

To avoid these drawbacks, we propose to use this additional information to select the right solution inside the set of solutions of the forward kinematics process. This is achieved through a $\mathcal{FK}_{lsq}$ process (based on the minimization of the least square error for the 4 legs) used to determine $\bar{P}_k, \bar{R}_k$ and by using the cost function based on the following calibration equations:

$$S_k(\mathbf{p}, \mathbf{q}_k) = \sum_{i=1}^{4} \left\| \|\bar{P}_k + \bar{R}_k.b_i - a_i\|^2 - l_{k,i}^2 \right\|^2 \tag{7}$$

And as the amplitude of the error associated with different sensors may be different, a weighting of the least-square forward kinematics criterion is necessary.

3.3 Root-matching approach

The root-matching approach is an exact method based on the comparison between the pose calculated by a forward kinematics process from the measurements and the kinematics parameters and, generally, some other information involving the generalised coordinates. This information may be provided by an external measurement device, or via a mechanical constraint. In the current exemple,the additional redundant sensor provides another way to calculate the pose parameters. If, for the k–th configuration, we denote by $\bar{P}$, $\bar{R}$ (resp. $\tilde{P}$, $\tilde{R}$ the position and orientation defining the pose computed through a forward kinematics resolution of the robot ignoring the 4–th leg (resp. the 3 leg), then the calibration equations are defined as the difference between $\bar{P}$ and $\tilde{P}$, and between $\bar{R}$ and $\tilde{R}$.

Two difficulties are linked to this approach.

- The coordinates of the pose parameters are not homogeneous and are mixing position and orientation units[1] and, in some cases, it is difficult to change the parameterization to avoid this drawback and to give a sense to the equations and to the criterion independently of any unit.

- There is a difference of magnitude between the error associated with internal sensors, the error associated with additional sensors or with mechanical constraints. This requires application of a scaling factor to these errors based on a statistical analysis of their variance.

In the case of the 4-RPR planar robot, we have seen in section 2.5 that the forward kinematics model can be symbolically manipulated and simplified to eliminate the position parameters. Then we are able to re-formulate the calibration equations as a distance constraint between the roots of the corresponding polynomial U_1 and U_2 in $t = \tan\frac{\theta}{2}$ for each of the n configurations.

The *resultant* (or *eliminant*) is an algebraic tool for studying the common roots of two or more polynomials. It is a polynomial expressed either as a determinant or as a non-trivial divisor of a determinant; see e.g. Nielsen and Roth (1995); Emiris and Mourrain (1999) and their references for details.

To eliminate the variable t, we compute the resultant of U_1 and U_2 and we get the calibration equations as determinants of so-called elimination matrices. Several constructions of such a matrix are proposed, namely a Sylvester or Bezout matrix, or after expression of the polynomials in a Bernstein basis, another form of a Bezout (or Dixon) matrix Bini and Gemignani (2004).

The size of these matrices are 12×12 for Sylvester, 6×6 for Bezout. The size and the degree of each coefficient of the matrix participate to the numerical stability of the evaluation of the calibration equations. Two other different matrices of elimination using directly the initial equations provided by the system 3 to eliminate directly the generalized coordinates x, y and θ can be computed by multidimensional elimination techniques (such as a Bezout or a sparse resultant matrix): in the following section, we use the Maple package *Multires* provided in Bus et al. (2003) for that purpose. The size of the computed matrices are 57×57 for the sparse resultant matrix, and 14×12 for the multidimensional Bezout matrix.

Note that we are not really interested in computing the whole elimination matrix but only in the localisation of the values of parameters for which the rank of the elimination matrix is decreasing: these values correspond to our case of a redundant robot. For that purpose, the best suited criterion, from a numerical stability point of view is the product of the sigular values of the matrix.

One interesting question is what is the number of solutions of this calibration problem. In the current case, where the measurement of the position and/or the orientation is not available, the solution is not unique. If we denote by $P_k = \hat{P}_k, R_k = \hat{R}_k, a = \hat{a}, b = \hat{b}$ a solution of the problem, then the values

[1] Many papers in robotics deal with this problem in calibration see Iurascu C.C. (2003).

$P_k = \hat{R}_k^T.\hat{P}_k, R_k = \hat{R}_k^T, a = \hat{b}, b = \hat{a}$ are also a solution, and the same negative values as well. This remark is valid for frame definition given in 4, it allows us to define a smoother version of the calibration criterion by considering the aggregated residual at a point and at its symetric point.

4 Simulations

We compare the different approaches of the calibration of the 4-RPR planar robot. These coordinates are considered as the actual values of the robot $\mathcal{P}_a$. The range of variation of the leg length is between 20 and 30.

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
x	0	40	40	0	0	20	20	0
y	0	0	40	40	0	0	10	10

Table 1: $\mathcal{P}_a$ Coordinates of the attachment point of the 4-RPR planar robot in base (a_i) and mobile (b_i) frames.

We select 32 configurations of measurement on the boundary of the workspace. The corresponding leg lengths are denoted by $\hat{L}_{i,k}$ and thecorresponding poses are denoted by $\hat{\mathcal{X}}_k = [\hat{P}_k, \hat{R}_k]$. These poses are chosen because they improve the observability index associated to the identification Jacobian – see Patel and Ehmann (2000); Daney et al. (2005).

The aim of the simulation is to compare the different approaches: first, to check the convergence of the algorithms without any measurement noise associated to the measurement of the leg length; second, to check the robustness of the identification with respect to measurement noise.

We distinguish the different approaches using different notation for the calibrating equations: by $\mathcal{M}_B$ we denote the basic method described in section 3.1, by $\mathcal{M}_C$ we denote the classic method – see equation (6) and by $\mathcal{M}_L$ we denote the least-squares approach – see equation (7). Finally, for the resultant approach, we denote by $\mathcal{M}_R$ the method that uses the multidimensional Bezout matrix [2]

4.1 Without measurement noise

We test the convergence of the methods to a solution ($\mathcal{P}_{sol}$) for different initial estimation of the kinematics parameters $\hat{\mathcal{P}}$. These estimation have been randomly selected - uniformly distributed - inside given ranges of values. However, the methods $\mathcal{M}_B, \mathcal{M}_C, \mathcal{M}_L$ also need an initial estimation of the measurement poses noted $\mathcal{X}_k^0 = [x_k^0, y_k^0, \theta_k^0]$. Then, the index $1, 2, 3$ denote 3 amplitude of error for $\mathcal{X}_k^0$:

- For the methods $\mathcal{M}_{B1}, \mathcal{M}_{C1}, \mathcal{M}_{L1}$, the initial error is given such as $x_k^0 \in [\hat{x}_k \pm 1]$ unity, $y_k^0 = [\hat{y}_k \pm 1]$ unity, $\theta_k^0 = [\hat{\theta}_k \pm 0.1]$ radian.
- For the methods $\mathcal{M}_{B2}, \mathcal{M}_{C2}, \mathcal{M}_{L2}$, the initial error is given such as $x_k^0 \in [\hat{x}_k \pm 5]$ unity, $y_k^0 = [\hat{y}_k \pm 5]$ unity, $\theta_k^0 = [\hat{\theta}_k \pm 0.5]$ radian.
- For the methods $\mathcal{M}_{B3}, \mathcal{M}_{C3}, \mathcal{M}_{L3}$, the initial estimations of x_k^0 are chosen so as to correspond to the center of the workspace.

[2]We omit the results provided by other kind of elimination matrices since they give worse results. The comparison of different elimination techniques is beyond the realm of this paper.

We perform every experiment 100 times, with different values for the initial estimation of the kinematics parameters (the range of the norm of the error $\hat{\mathcal{P}}$ is given in column 1). The percentage of convergence are given in 2.

err. init. est.	$\mathcal{M}_{B1}$	$\mathcal{M}_{B2}$	$\mathcal{M}_{B3}$	$\mathcal{M}_{C1}$	$\mathcal{M}_{C2}$	$\mathcal{M}_{C3}$	$\mathcal{M}_{L1}$	$\mathcal{M}_{L2}$	$\mathcal{M}_{L3}$	$\mathcal{M}_{R}$
$[0, 5]$	100	98	83	78	2	0	100	100	85	53
$[5, 10]$	100	94	88	70	0	0	100	100	59	35
$[10, 15]$	100	85	64	-	-	-	100	100	42	-
$[15, 20]$	90	80	20	-	-	-	100	95	22	-
$[20, 30]$	81	78	-	-	-	-	88	90	-	-

Table 2: Comparison of the convergence percentage of the different approaches without measurement noise

Table 2 provides many interesting results. First of all, the classical method, that corresponds to the numerical forward kinematics approach, is not efficient. This is so, because it convergences to not actual measurement poses. Hence, the multiple solution of the forward kinematics is then problematic for the calibration of a parallel robot. The approach using a least-square forward kinematics is much more robust to the error on the estimation of the kinematics parameters. This is not surprising, since this model admits, generically, one solution. The classical method, where the robot poses are also determined by the identification process provides very good results. This leads to the question of whether it is interesting to eliminate the pose-dependent parameters. The resultant approach provides quite good results, as well. This is due to the reason that there exists an attractive minimum at infinity. We plot, in figure 2(a), the criterion equation as a function of (a scaling of) the actual kinematics parameters $C(s \times \hat{a}, s \times \hat{b})$ (the axes values are given in $\log_{10}$). Note that this criterion, even if it is here computed numerically through an SVD decomposition, is somehow equivalent to an algebraic function of the parameters of degree 66. We remark that for $s \to 0$ or $s \to +\infty$ the criterion converges to a minimum. The zoom, given in figure 2(b), indicates exactly this, since if we scale the platform about 20% for our initial estimation, the approach tends to diverge. This explains the result of method $\mathcal{M}_R$.

Remarks: We have tested the method for a robot as defined by Husty in Husty M. (2001). For all methods, the calibration of this robot is problematic. We think that it is interesting to study robots closed to this special one as identification of parameters in these cases is very unstable numerically.

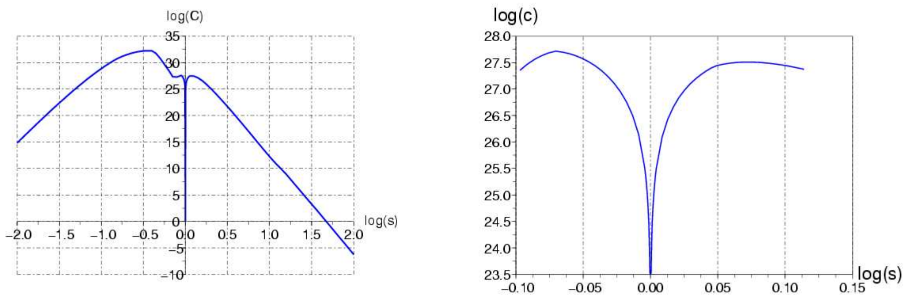
4.2 With measurement noise

To test the robustness of the approaches as a function of the measurement error, we choose randomly an initial estimation of the kinematics parameters $\mathcal{P}^0$. The norm of the error $\|\mathcal{P}^0 - \hat{\mathcal{P}}\|$ is equal to 1.3 units.

We add to leg length $L_{i,k}$ an error uniformly distributed in a range equal to $[-0.001, +0.001]$, $[-0.01, +0.01]$ or $[-0.1, +0.1]$. We apply method $\mathcal{M}_{B2}$, $\mathcal{M}_{L2}$ and $\mathcal{M}_R$ which identifies the kinematics parameters, denoted $\mathcal{P}_c$. Table 3 provides

- the percentage of experiments where the identification process provides a better estimation of $\hat{\mathcal{P}}$ (noted **SC** $(\%)$). In this case, the mean and the standard deviation of the improvement[3] is computed.

[3]The percentage of improvement is defined as $\|\mathcal{P}^0 - \hat{\mathcal{P}}\| - \|\mathcal{P}_c - \hat{\mathcal{P}}\| / \|\mathcal{P}^0 - \hat{\mathcal{P}}_c\| \times 100$



(a) General view

(b) Zoom on interesting part

Figure 2: Criterion given by a resultant approach as a function of a scaling of the kinematics parameters.

- the percentage of experiments where the identification process provides a worst estimation of $\hat{\mathcal{P}}$ (noted **NSC** (%)).

err. on	$\mathcal{M}_{B2}$				$\mathcal{M}_{L2}$				$\mathcal{M}_{R}$			
$L_{i,k}$	**SC**	Mean	Std	**NSC**	**SC**	Mean	Std	**NSC**	**SC**	Mean	Std	**NSC**
± 0.001	95.	93.4	3.5	5	100.	93.0	3.93	0.	85.	48.76	22.0	15.
± 0.01	97.	93.0	3.4	3	100.	92.8	4.1	0.	85	49.2	21.4	15.
± 0.1	77.	44.75	23.16	23.	76.	44.84	23.28	24.	1.	10.36	0.	99.

Table 3: Calibration results with different level of measurement noise

Approach $\mathcal{M}_{L2}$ gives the best results, since the convergence of $\mathcal{M}_{B2}$ tends to be more sensible to measurement noise. Method $\mathcal{M}_R$ provides the worst results. This is due, in part, to the high degree of the algebraic constraints involved; a remedy for this has been proposed in Daney and Emiris (2001).

5 Conclusion

This paper propose to compare different approaches to eliminate the pose-dependent parameters. Due to the multiple solutions of the forward kinematics model, the classical approach does not converge to the solution in a lot of cases. In practice, the difficulties appear if the initial estimations of the kinematics parameters are not close to the actual values. For the calibration of a machine tools (PKM), for example, the estimations are good enough to avoid this drawback. However, we have seen that a least-square estimation of a solution of the forward kinematics model improves the convergence and decreases the sensibility of the result to the measurement noise.

The basic method, which does not eliminate the pose-dependent parameters, provides also and surprisingly some good results. One possible explanation is that the number of unknowns (106 in our simulation) is small enough to be tractable. But this is interesting to note that when the number of parameters is not too large, it may not be worse to eliminate them. And we plan to study the calibration of the Gough platform with one additional leg based on this approach.

The method using a resultant-based elimination is less efficient. We have observed how it is difficult to implement it, but we did not explore all possible improvements. Its main advantage is that it requires no initial estimation of the measurement configurations. That may be interesting for a particular class of applications – see Tadokoro et al. (1999). The extension of this method to the 3-dimensional case of the Gough platform is theoretically possible because Innocenti in Innocenti (1998) provides the required elimination matrix. However, the degree of the resulting polynomial may be too high to allow for all required computation.

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