

Truncated Boomerang Attacks and Application to AES-based Ciphers

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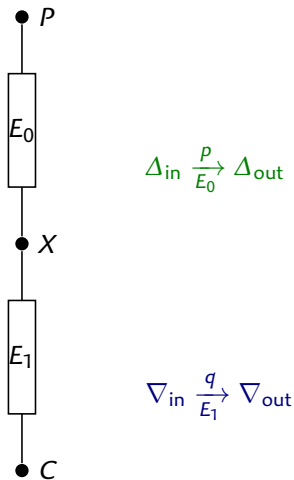
The AES

- ▶ AES is the **most widely used block cipher** today
 - ▶ Designed in 1999
 - ▶ Selected by NIST
- ▶ Round function and reduced versions **reused in many context**
 - ▶ Hash function: Grøstl (SHA-3 finalist), LED, ECHO
 - ▶ Stream cipher: LEX
 - ▶ MACs: Alpha-MAC
 - ▶ Tweakable block ciphers: Deoxys (CAESAR portfolio), KIASU, TNT
 - ▶ AEAD: Aegis (CAESAR portfolio), Tiaoxin
- ▶ Need **cryptanalysis** to evaluate security
 - ▶ New and old attack techniques
 - ▶ Many recent results!

[Daemen & Rijmen]
[FIPS 197]

The Boomerang Attack

[Wagner, FSE'99]



- ▶ Combine **two short differentials** instead of using a long one.

- ▶ $E = E_1 \circ E_0$

- ▶ $\Delta_{\text{in}} \xrightarrow{\frac{P}{E_0}} \Delta_{\text{out}}$

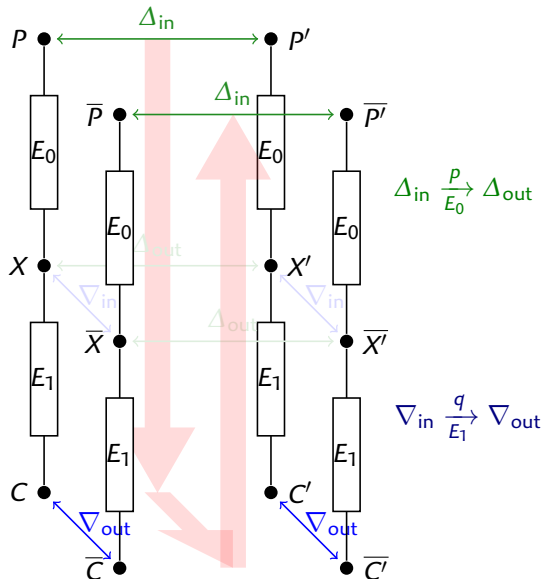
- ▶ $\nabla_{\text{in}} \xrightarrow{\frac{q}{E_1}} \nabla_{\text{out}}$

- ▶ Uses an **encryption** oracle and **decryption** oracle

- ▶ Adaptive attack

- ▶ Build quartets

Boomerang Quartet



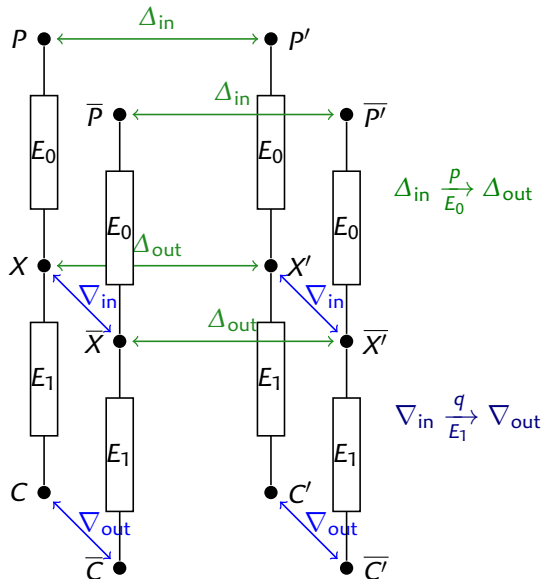
- 1 $P \leftarrow \$, \quad P' = P + \Delta_{in}$
- 2 $C = E(P), \quad C' = E(P')$
- 3 $\bar{C} = C + \nabla_{out}, \quad \bar{C}' = C' + \nabla_{out}$
- 4 $\bar{P} = E^{-1}(\bar{C}), \quad \bar{P}' = E^{-1}(\bar{C}')$
- 5 Check if $\bar{P} + \bar{P}' = \Delta_{in}$

Probability of returning: $p_b = p^2 q^2$

- ▶ $\Pr[X + X' = \Delta_{out}] = p$
- ▶ $\Pr[X + \bar{X} = \nabla_{in}] = q$
- ▶ $\Pr[X' + \bar{X}' = \nabla_{in}] = q$
- ▶ If this holds, then $\bar{X} + \bar{X}' = \Delta_{out}$
- ▶ $\Pr[\bar{P} + \bar{P}' = \Delta_{in}] = p$

Distinguisher if $p_b \gg 2^{-n}$

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Distinguisher if $p_b \gg 2^{-n}$

Our results

1 Revisiting boomerang with **truncated differentials**

[Wagner, FSE'99]

- ▶ Use of structures: plaintext ciphertext
- ▶ Statistical distinguishers and key-recovery
- ▶ Generic formula for complexity

2 **Improving** boomerang attack on 6-round AES

[Biryukov, AES'04]

- ▶ Key-recovery with complexity 2^{61} (improved from 2^{71})
- ▶ Key-recovery with secret S-Boxes
- ▶ 6-round statistical distinguisher ("key-independent")

3 **Best attacks** on several AES-based tweakable block ciphers

- ▶ KIASU
- ▶ TNT-AES
- ▶ Deoxys

[Jean, Nikolić & Peyrin, AC'14]

[Bao, Guo, Guo & Song, EC'20]

[Jean, Nikolić & Peyrin, AC'14]

Outline

Introduction

Truncated Boomerang Distinguisher

Truncated Boomerang Key-recovery

Applications

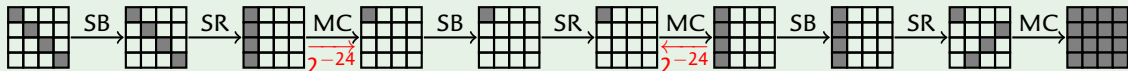
Conclusion

Truncated differential cryptanalysis

- ▶ **Generalisation** of differential cryptanalysis
- ▶ **Truncate information** about differences, (e.g. active/inactive bytes)
- ▶ Set of input/output differences: $\mathcal{D}_{\text{in}}, \mathcal{D}_{\text{out}}$
- ▶ $\vec{p} = \text{Avg}_{\Delta_{\text{in}} \in \mathcal{D}_{\text{in}}} \Pr [E(P) + E(P + \Delta_{\text{in}}) \in \mathcal{D}_{\text{out}}]$
- ▶ $\vec{p} = \text{Avg}_{\Delta_{\text{out}} \in \mathcal{D}_{\text{out}}} \Pr [E^{-1}(P) + E^{-1}(P + \Delta_{\text{out}}) \in \mathcal{D}_{\text{in}}]$
- ▶ $\frac{\vec{p}}{|\mathcal{D}_{\text{out}}|} = \frac{\vec{p}}{|\mathcal{D}_{\text{in}}|} = \text{Avg}_{\Delta_{\text{in}} \in \mathcal{D}_{\text{in}}, \Delta_{\text{out}} \in \mathcal{D}_{\text{out}}} \Pr [E(P) + E(P + \Delta_{\text{in}}) = \Delta_{\text{out}}]$

[Kundsen, FSE'94]

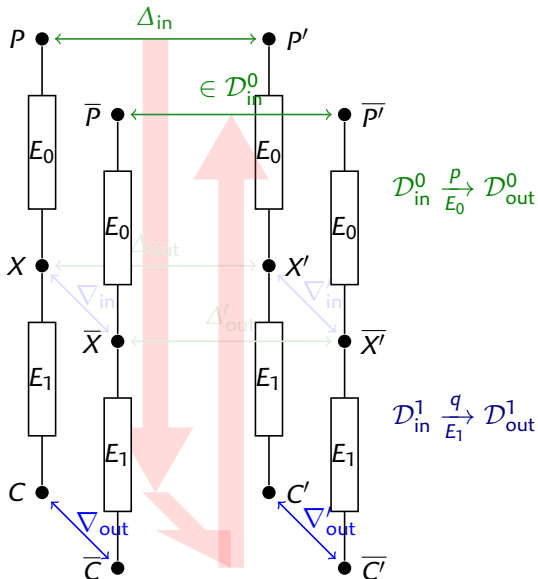
Example: 3-round AES truncated trail



▶ $|\mathcal{D}_{\text{out}}| = |\mathcal{D}_{\text{in}}| = 2^{32}$

▶ $\vec{p} = \vec{p} = 2^{-24}$

Truncated Boomerang Quartet

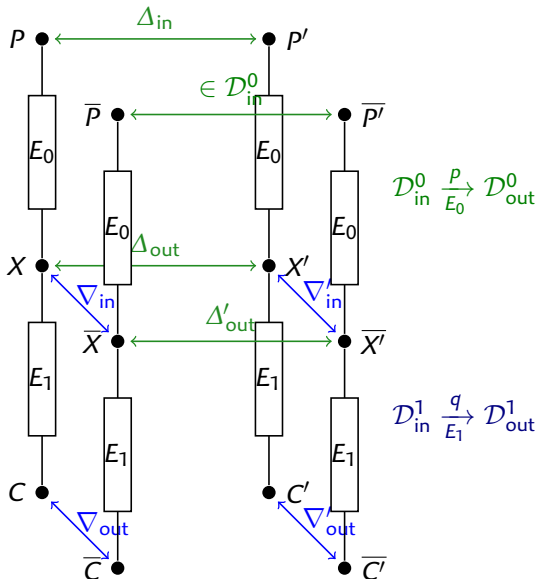


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 - 3 $\bar{C}' = C' + \nabla'_{out}$, $\nabla'_{out} \in \mathcal{D}_{out}^1$
 - 4 Check if $\bar{P} + \bar{P}' \in \mathcal{D}_{in}^0$
- Note: $\nabla_{out} \neq \nabla'_{out}$

Probability of returning: $p_b = \bar{p} \cdot \tilde{p} \cdot \tilde{q}^2 \cdot r$

- $\Pr[X + X' \in \mathcal{D}_{out}^0] = \bar{p}$
- $\Pr[X + \bar{X} \in \mathcal{D}_{in}^1] = \tilde{q}$
- $\Pr[X' + \bar{X}' \in \mathcal{D}_{in}^1] = \tilde{q}$
- $\Pr[\bar{X} + \bar{X}' \in \mathcal{D}_{out}^0] = r \geq |\mathcal{D}_{in}^1|^{-1}$
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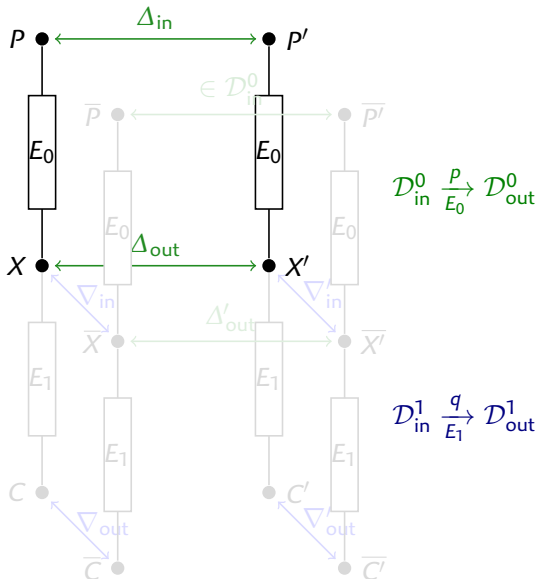


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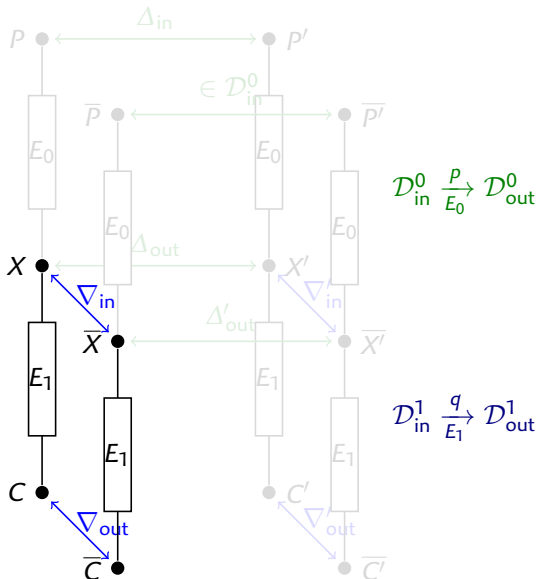


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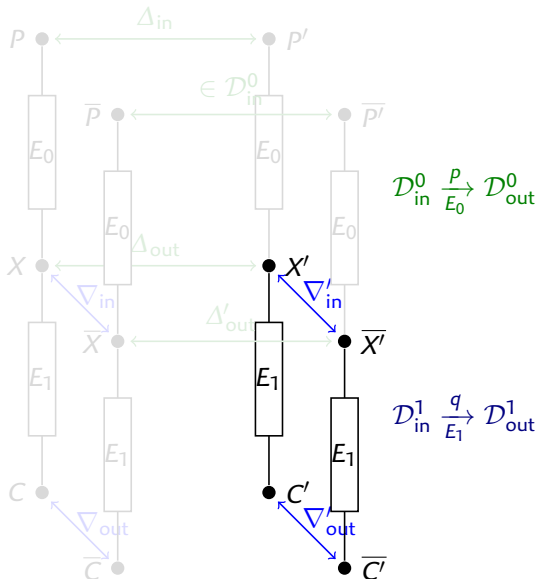


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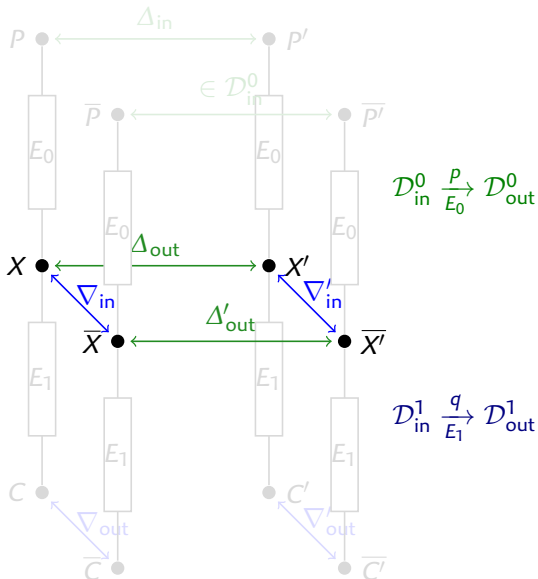


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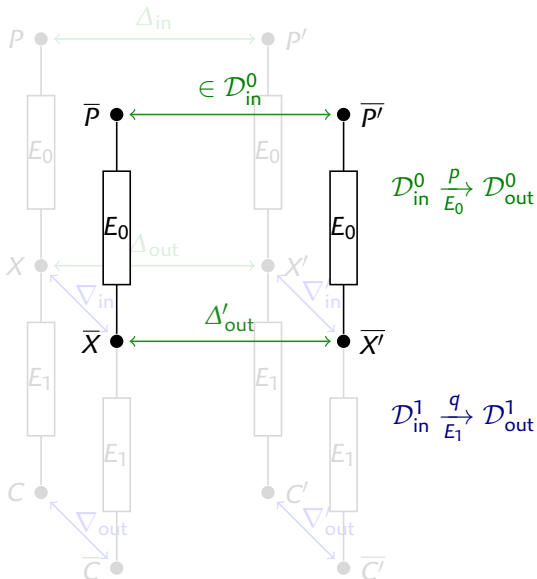


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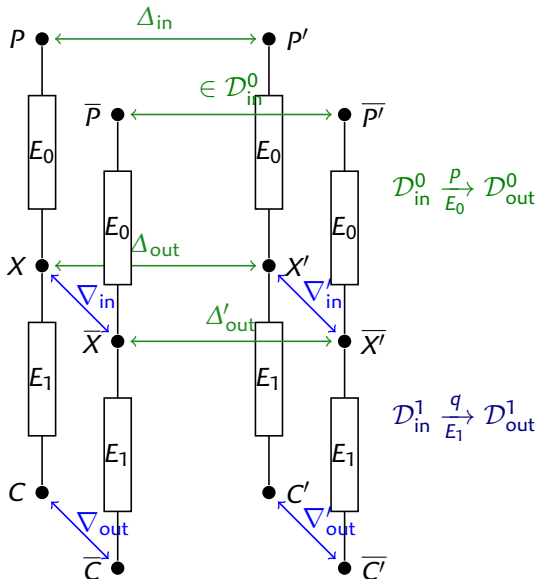


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Using structures

- ▶ Assuming $\mathcal{D}_{\text{in}}^0$ is a vector space

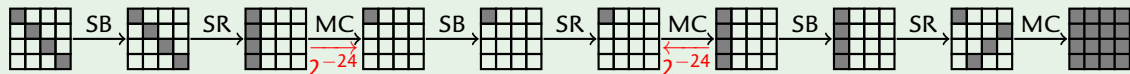
- 1 Start with a **structure of plaintext**
 - 2 Build a **structure for each ciphertext**
- ▶ Total structure size $|\mathcal{D}_{\text{in}}^0| \cdot |\mathcal{D}_{\text{out}}^1|$
 - ▶ $|\mathcal{D}_{\text{in}}^0|$ encryption queries
 - ▶ $|\mathcal{D}_{\text{in}}^0| \cdot |\mathcal{D}_{\text{out}}^1|$ decryption queries
 - ▶ $|\mathcal{D}_{\text{in}}^0|^2 \cdot |\mathcal{D}_{\text{out}}^1|^2$ candidate quartets

Truncated Boomerang Distinguisher

- 1 Choose a random P_0
 - ▶ Define $P_i = P_0 + i$ for $i \in \mathcal{D}_{\text{in}}^0$
- 2 Query $C_i = E(P_i)$
 - ▶ Define $\bar{C}_i^j = C_i + j$ for $j \in \mathcal{D}_{\text{out}}^1$
- 3 Query $\bar{P}_i^j = E^{-1}(\bar{C}_i^j)$
- 4 Count pairs with $\bar{P}_i^j + \bar{P}_{i'}^{j'} \in \mathcal{D}_{\text{in}}^0$
- 5 If needed, repeat with new P_0

Example: 6-round AES boomerang

3-round AES truncated trail for E_0 and E_1



$$\blacktriangleright |\mathcal{D}_{\text{out}}^0| = |\mathcal{D}_{\text{in}}^0| = 2^{32}$$

$$\blacktriangleright |\mathcal{D}_{\text{out}}^1| = |\mathcal{D}_{\text{in}}^1| = 2^{32}$$

$$\blacktriangleright r = |\mathcal{D}_{\text{in}}^1|^{-1} = 2^{-32}$$

$$\blacktriangleright \vec{p} = \vec{p} = 2^{-24}$$

$$\blacktriangleright \vec{q} = \vec{q} = 2^{-24}$$

$$\blacktriangleright p_b = \vec{p} \cdot \vec{p} \cdot \vec{q}^2 \times r = 2^{-128}$$

- ▶ One structure has $|\mathcal{D}_{\text{in}}^0| \cdot |\mathcal{D}_{\text{out}}^1| = 2^{64} \overline{P}_i^j$
 - ▶ 2^{127} pairs: candidate quartets
 - ▶ $2^{127} \cdot p_b = 1/2$ good quartets
 - ▶ $2^{127} \cdot 2^{-96} = 2^{31}$ returning quartets: wrong quartets

▶ Most returning quartets are fake positive

▶ Detect signal with $\ggg 2^{32}$ structures: $T = D = \mathcal{O}(2^{96})$

Analysis

- ▶ Starting from S structures of size $|\mathcal{D}_{\text{in}}^0| \cdot |\mathcal{D}_{\text{out}}^1|$
- ▶ $Q = S \times |\mathcal{D}_{\text{in}}^0|^2 \cdot |\mathcal{D}_{\text{out}}^1|^2$ candidate quartets
- ▶ Boomerang probability $p_b = \vec{p} \cdot \vec{p} \cdot \vec{q}^2 \cdot r$
- ▶ Random probability $p_{\$} = |\mathcal{D}_{\text{in}}^0|/2^n$
- ▶ Signal to noise $\sigma = p_b/p_{\$}$
- ▶ $Q \cdot p_b$ good quartets
- ▶ $Q \cdot p_{\$}$ wrong quartets

If $\sigma \gg 1$

- ▶ A few good quartets are sufficient
- ▶ $Q = \mathcal{O}(1/p_b)$ quartets needed

If $\sigma \ll 1$

- ▶ More wrong quartets than good
- ▶ $Q = \mathcal{O}(1/\sigma p_b)$ quartets needed

- ▶ Time and data complexity

$$T = D = \frac{2Q}{|\mathcal{D}_{\text{in}}^0| \cdot |\mathcal{D}_{\text{out}}^1|}$$

Outline

Introduction

Truncated Boomerang Distinguisher

Truncated Boomerang Key-recovery

Applications

Conclusion

Key recovery

- ▶ **Usual approach:** add rounds before/after distinguisher
 - ▶ More rounds, higher complexity than distinguisher
- ▶ **Our approach:** extract key information from right pairs
 - ▶ Same number of rounds, lower complexity than distinguisher
- ▶ Roughly equivalent, but easier to analyse with generic formulas

If $\sigma \gg 1$

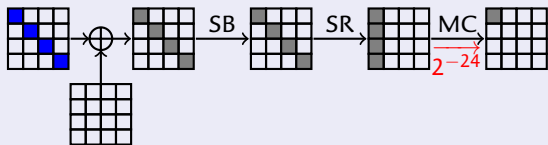
- ▶ Collect one good quartet
- ▶ (P, P') and (\bar{P}, \bar{P}') follow E_0 trail
- ▶ (C, \bar{C}) and (C', \bar{C}') follow E_1 trail
 - ▶ This is only true for a subset of keys
 - ▶ Recover ℓ candidates for a κ -bit key

If $\sigma \ll 1$

- ▶ Collect many quartets
- ▶ Assume quartets are good
- ▶ (P, P') and (\bar{P}, \bar{P}') follow E_0 trail
- ▶ (C, \bar{C}) and (C', \bar{C}') follow E_1 trail
 - ▶ This is only true for a subset of keys
 - ▶ Recover ℓ candidates for a κ -bit key
- ▶ Use counters for key candidates
- ▶ Right key suggested more frequently

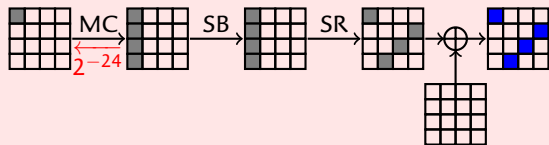
Example: 6-round AES boomerang

First round



- ▶ Plaintext is known
- ▶ Recover candidates for k_0 diagonal
 - ▶ Given (P, P') , 2^8 candidates
 - ▶ Given (\bar{P}, \bar{P}') , 2^8 candidates
 - ▶ 2^{-16} candidates in intersection

Last round



- ▶ Ciphertext is known
- ▶ Recover candidates for k_6 anti-diagonal
 - ▶ Given (C, \bar{C}) , 2^8 candidates
 - ▶ Given (C', \bar{C}') , 2^8 candidates
 - ▶ 2^{-16} candidates in intersection

- ▶ On average $\ell = 2^{-32}$ candidates for $\kappa = 64$ bits of key
- ▶ With **S structures**: $S \times 2^{64}$ elements \bar{P}_i^j , $S \times 2^{127}$ pairs, $S \times 2^{31}$ returning quartets
 - ▶ $S \times 2^{31}$ fake positives $\rightarrow S \times 2^{31} \times 2^{-32} = S/2$ wrong keys suggestions
 - ▶ $S \times 1/2$ right quartet $\rightarrow S \times 1/2 \times 1 = S/2$ correct keys suggestions
- ▶ High probability of succes with 8 structures ($D = T = 2^{67}$)

Analysis

- ▶ Starting from S structures of size $|\mathcal{D}_{in}^0| \cdot |\mathcal{D}_{out}^1|$
- ▶ $Q = S \times |\mathcal{D}_{in}^0|^2 \cdot |\mathcal{D}_{out}^1|^2$ candidate quartets, $Q \cdot p_{\$}$ returning quartets
- ▶ $Q \cdot p_b$ good quartets
 - ▶ 1 suggestion for right key
 - ▶ ℓ suggestions for wrong key, $\ell \times 2^{-\kappa}$ hits for each
- ▶ $Q \cdot p_{\$}$ fake positives
 - ▶ ℓ suggestions for wrong key, $\ell \times 2^{-\kappa}$ hits for each
- ▶ Improved signal to noise $\tilde{\sigma} = p_b / p_{\$} \times 2^{\kappa} / \ell$

If $\tilde{\sigma} \gg 1$

- ▶ A few good quartets are sufficient
- ▶ $Q = \mathcal{O}(1/p_b)$ quartets needed

- ▶ Time and data complexity

$$T = D = \frac{2Q}{|\mathcal{D}_{in}^0| \cdot |\mathcal{D}_{out}^1|}$$

If $\tilde{\sigma} \ll 1$

- ▶ More wrong quartets than good
- ▶ $Q = \mathcal{O}(1/\tilde{\sigma}p_b)$ quartets needed

6-round AES results

	Type	Data		Time	Ref
Distinguishers	Yoyo	$2^{122.8}$	ACC	$2^{121.8}$	[AC:RonBarHel17]
	Exchange attack	$2^{88.2}$	CP	$2^{88.2}$	[AC:BarRon19]
	Exchange attack	2^{84}	ACC	2^{83}	[EPRINT:Bardeh19]
	Truncated differential	$2^{89.4}$	CP	$2^{96.5}$	[ToSC:BaoGuoLis20]
	Truncated boomerang	2^{87}	ACC	2^{87}	New
Key-recovery	Square	2^{32}	CP	2^{71}	[FSE:DaeKnuRij97]
	Partial-sum	2^{32}	CP	2^{48}	[FSE:FKLSSWW00]
	Boomerang	2^{71}	ACC	2^{71}	[biryukov2004boomerang]
	Mixture	2^{26}	CP	2^{80}	[JC:BDKRS20]
	Retracing boomerang	2^{55}	ACC	2^{80}	[EC:DKRS20]
	Boomeyong	$2^{79.7}$	ACC	2^{78}	[ToSC:RahSahPau21]
	Truncated boomerang	2^{59}	ACC	2^{61}	New
Secret S-Box KR	Square	2^{64}	CP	2^{90}	[FSE:TKKL15]
	Truncated boomerang	2^{94}	ACC	2^{94}	New

Outline

Introduction

Truncated Boomerang Distinguisher

Truncated Boomerang Key-recovery

Applications

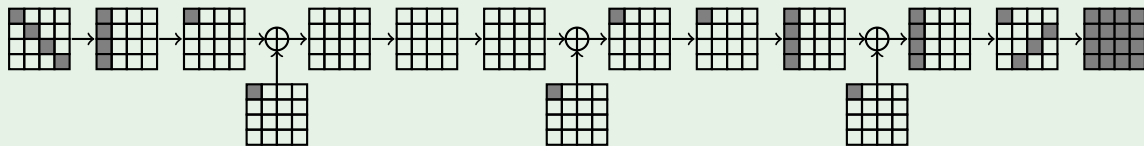
Conclusion

8-round boomerang on KIASU

- ▶ **KIASU**: AES-based tweakable block cipher
 - ▶ Tweak added on first 64 bits of state

[Jean, Nikolić & Peyrin, AC'14]

4-round truncated trail for KIASU



$$\vec{p} = 2^{-32}$$

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$$|\mathcal{D}_{\text{in}}^0| = 2^{32}$$

$$|\mathcal{D}_{\text{out}}^0| = 2^{32}$$

$$|\mathcal{D}_{\text{tw}}^0| = 2^8$$

- ▶ Evaluate complexity with generic formula

$$p_b = \vec{p} \cdot \vec{p} \cdot \vec{q}^2 \times |\mathcal{D}_{\text{in}}^1|^{-1} = 2^{-160}$$

$$\tilde{\sigma} = 2^{32}$$

$$p_w = |\mathcal{D}_{\text{in}}^0| / 2^n \times \ell \times 2^{-\kappa} = 2^{-192}$$

$$Q = \mathcal{O}(2^{160})$$

$$D = \mathcal{O}(2^{80})$$

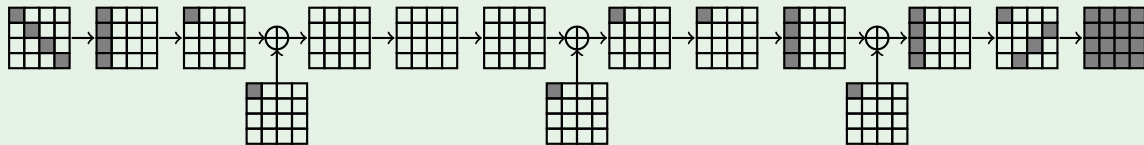
- ▶ Previous best attack: boomerang with complexity 2^{103}

8-round boomerang on KIASU

- ▶ **KIASU**: AES-based tweakable block cipher
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- ▶ Previous best attack: boomerang with complexity 2^{103}

Deoxys

- ▶ AES-based Tweakable block cipher, CAESAR portfolio [Jean, Nikolić & Peyrin, AC'14]
- ▶ Best attacks: boomerangs built with MILP model
 - ▶ Key-recovery typically added afterwards

Our results

- ▶ **MILP model** with truncated boomerang framework (model truncated trails)
- ▶ Integrate **key recovery**: optimize data complexity (parameters given by trail)

Model	Rnd		Previous			New			
			Data	Time	Mem	Data	Time	Mem	
RTK2	9	B	2^{98}	2^{112}	2^{17}	B	$2^{55.2}$	$2^{55.2}$	$2^{55.2}$
	10	B	$2^{98.4}$	$2^{109.1}$	2^{88}	B	$2^{94.2}$	$2^{95.2}$	$2^{94.2}$
	11	R	$2^{122.1}$	$2^{249.9}$	$2^{128.2}$	B	2^{129}	$2^{223.9}$	2^{129}
RTK3	11	B	2^{100}	2^{100}	2^{17}	B	$2^{32.7}$	$2^{32.7}$	$2^{32.7}$
	12	B	2^{98}	2^{98}	2^{64}	B	$2^{67.4}$	$2^{67.4}$	2^{65}
	13	R	$2^{125.2}$	$2^{186.7}$	2^{136}	B	$2^{126.7}$	$2^{170.2}$	$2^{126.7}$
	14	R	$2^{125.2}$	$2^{282.7}$	2^{136}	B	2^{129}	$2^{278.8}$	2^{129}

TNT-AES

- ▶ AES-based tweakable block cipher
- ▶ Uses 6-round AES as building block R
 - ▶ $\tilde{E} : T, P \mapsto R_2(T + R_1(T + R_0(P)))$
- ▶ Build **boomerang quartets for middle layer** using tweak differences
 - ▶ Only one usable return difference
 - ▶ No structures on ciphertext side
- ▶ **First attack based on a 6-round distinguisher**

Rounds	Type	Data		Time	Ref
-5-	Boomerang (dist.)	2^{126}	ACC	2^{126}	[EC:BGGS20]
5-*-*	Impossible differential (KR)	$2^{113.6}$	CP	$2^{113.6}$	[AC:GGLS20]
--*	Generic (dist.)	$2^{99.5}$	CP	$2^{99.5}$	[AC:GGLS20]
-5-	Truncated boomerang (dist.)	2^{76}	ACC	2^{76}	New
5-5-*	Truncated boomerang (KR)	2^{87}	ACC	2^{87}	New
-6-	Truncated boomerang (dist.)	$2^{127.8}$	ACC	$2^{127.8}$	New

Conclusion

- 1 Analysis of truncated bommerang attacks
 - ▶ Use of structures
 - ▶ Generic formulas for data complexity
- 2 Revisiting boomerangs on 6-round AES
 - ▶ Competitive with recently proposed 6-round attacks
 - ▶ Statistical distinguisher (“key-independent”)
 - ▶ Key recovery
 - ▶ Key-recovery with secret S-Boxes
- 3 Applications
 - ▶ Best attack on KIASU
 - ▶ Marginal distinguisher on TNT-AES
 - ▶ First application of a 6-round distinguisher
- 4 Implementation as a MILP model
 - ▶ New results on Deoxys