Practical Key Recovery Attack against Secret-IV Edon-$\mathcal{R}$

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Abstract. The SHA-3 competition has been organized by NIST to select a new hashing standard. Edon-$\mathcal{R}$ was one of the fastest candidates in the first round of the competition. In this paper we study the security of Edon-$\mathcal{R}$, and we show that using Edon-$\mathcal{R}$ as a MAC with the secret-IV or secret-prefix construction is unsafe. We present a practical attack in the case of Edon-$\mathcal{R}$256, which requires 32 queries, 2^{30} computations, negligible memory, and a precomputation of 2^{52}. The main part of our attack can also be adapted to the tweaked Edon-$\mathcal{R}$ in the same settings: it does not yield a key-recovery attack, but it allows a selective forgery attack.

This does not directly contradict the security claims of Edon-$\mathcal{R}$ or the NIST requirements for SHA-3, since the recommended mode to build a MAC is HMAC. However, we believe that it shows a major weakness in the design.

Key words: Hash functions, SHA-3, Edon-$\mathcal{R}$, MAC, secret IV, secret prefix, key recovery.

1 Introduction

In 2005, a team of researchers led by X. Wang produced breakthrough attacks against many widely used hash functions, including MD5 [12] and SHA-1 [11]. This has led NIST to call for a new hash function design, and to launch the SHA-3 competition [7]. This competition has focused the attention of many cryptographers, and NIST received 64 submissions. 51 designs were accepted to the first round.

Edon-$\mathcal{R}$ was one of the fastest candidates in the first round of the competition. It has received some attention from the cryptographic community, resulting in various attacks on the compression function. There is also a preimage attack on the full hash function, but it requires of huge amount of memory making it debatable.

In this paper we show a new attack on Edon-$\mathcal{R}$, when used in the secret-IV or secret-prefix MAC construction. This mode of operation is not claimed to be secure by the designers, but our attack has no memory requirement, and is even practical attack, while previous attacks are largely theoretical. Our approach is similar to the one followed by Wang *et al.* who studied a similar MAC used with SHA-1 [10]: we use a non-standard MAC to show weaknesses of the hash function. Note that attacks on hash-based MACs are usually harder to build than attacks on the hash function itself because part of the state is unknown.

Our attack was devised during the first round of the competition and it has been made public before the selection of the second round candidates. Since then, NIST selected 14 candidates for the second round of the competition, based on the cryptanalytic results available at that time. EDON-$\mathcal{R}$ was not selected for the second round.

1.1 MAC Constructions

A Message Authentication Code (MAC) is a symmetric signature algorithm. The sender and the receiver share a secret key k, and each message M is sent together with a short tag $\mathrm{MAC}_k(M)$. The receiver recomputes the tag on his end, and checks whether the tag is correct. It should be hard for an adversary to forge a message with a valid tag without knowing the secret key k. In this paper we consider chosen message attacks, where the adversary has access to a MAC oracle and can ask for the MAC of any message of his choice. He must then produce a forge for a new message. We consider the following attacks, from the strongest to the weakest:

Key recovery: After some interactions with the MAC oracle, the adversary outputs the key k.

Universal forgery: After some interactions with the MAC oracle, the adversary obtains enough information to compute the MAC of any message. This is usually achieved by recovering an equivalent key which allow to compute MACs as efficiently as the original key.

Selective forgery: The adversary is given a challenge message M^* (possibly in some prescribed set), and after some interaction with the MAC oracle, he has to produce the MAC of M^*. Of course, the adversary is not allowed to query the MAC of M^*.

Exitential forgery: After some interactions with the MAC oracle, the adversary produce a forge of a new message of his choice.

We expect a good MAC algorithm to be secure against all these attacks, even existential forgery. Complexity of generic attack against iterated MAC algorithm are given in Table 1.

In this paper, we study MAC algorithms based on EDON-$\mathcal{R}$. We consider two MAC constructions, the secret-IV construction and the secret-prefix construction:

$$\text{IV-MAC}_k(M) = \text{EDON-}\mathcal{R}_k(M)$$
$$\text{SP-MAC}_k(M) = \text{EDON-}\mathcal{R}(k\|M)$$

The secret-IV method uses the key as the initial value in the iterative construction of EDON-$\mathcal{R}$, while the secret-prefix method prepends the key to the message to be authenticated. Both constructions are quite similar, and the basic idea is

Table 1. Complexity of generic attacks against iterated MAC. The key-length n is assumed to be equal to the tag length, while m is the size of the inner state.

Attack	Complexity
Key recovery	2^n
Universal forgery	2^n
Selective forgery	2^n
Existential forgery	$\min(2^{m/2}, 2^n)$ [9]

to randomize the state of the hash function with the key before mixing the message into the state. The secret-IV construction is easier to analyse, but the secret prefix construction is more practical because it does not require a modification of the hash function; it can be used with any implementation of EDON-$\mathcal{R}$. For efficiency purpose, it is advisable to pad the key to a full block when using SP-MAC.

This kind of construction is used in some old protocols, like RFC2069 [2] (RFC2069 uses SP-MAC without padding the key to a full block). It is well known that those constructions are weak, because length extension attacks can be used for forgeries, but the key is not expected to leak. Moreover, EDON-$\mathcal{R}$ is a wide-pipe design, so the length extension issue does not apply. In fact, if the hash function is wide-pipe and the compression function is modeled as a random oracle, those constructions are provably secure [1]. Therefore, breaking EDON-$\mathcal{R}$ in this setting is expected be as hard a the generic complexities given in Table 1.

1.2 Road Map

Section 2 will describe EDON-$\mathcal{R}$ and discuss previous analysis. In Section 3, we show how to use a pair a related queries to gather information on both the input and the output of the compression function. The idea is similar to the length extension attack against Merkle-Damgård hash functions. This reduces the key-recovery problem to solving a small equation. In Section 4, we show how to solve this equation. We use simple linear algebra techniques to identify truncated differentials in the main operations of EDON-$\mathcal{R}$, and this leads to an attack with complexity $2^{5n/8}$ using only two queries to the MAC oracle. In Section 5 we use more queries to the MAC oracle to build more equations, and solve the equations using a guess-and-determine technique. This gives a very efficient attack, which is even practical in the case of EDON-$\mathcal{R}$224/256. Finally, in Appendix A, we show how to extend these results to attack against the secret-prefix construction, and attacks against MACs based on the tweaked version of EDON-$\mathcal{R}$.

2 Description of EDON-$\mathcal{R}$

EDON-$\mathcal{R}$ is a wide-pipe iterative design, based on a compression function $\mathcal{R}$, with a final truncation $\mathcal{T}$. The EDON-$\mathcal{R}$ family is based on two main designs:

EDON-$\mathcal{R}$256 uses 32-bits words, while EDON-$\mathcal{R}$512 uses 64-bit words. Let w denote the size of the words, and n denote the output size ($n = 8w$). We give a description of EDON-$\mathcal{R}$ where the variables are elements of $(\mathbb{F}_2^w)^8$, *i.e.*, 8-tuples of w-bit words. The compression function is based on a quasi-group operation $*$, which take two inputs X and Y in $(\mathbb{F}_2^w)^8$ and compute one output in $(\mathbb{F}_2^w)^8$. The quasi-group operation is just the sum of two permutations, and we will use a permutation based description of EDON-$\mathcal{R}$ in this paper:

$$\begin{aligned} X * Y &= \mu(X) + \nu(Y) \\ &= Q_0(R_0(P_0(X))) + Q_1(R_1(P_1(Y))) \end{aligned}$$

where

- $+$ is a component-wise addition modulo 2^w (w is the word size);
- μ and ν are the permutations defining $*$; we rewrite then with Q_i, R_i, P_i;
- P_0 and P_1 are linear over $\mathbb{Z}_{2^w}^8$, each output word is the sum of five inputs;
- R_0 and R_1 are component-wise rotations of w-bit words;
- Q_0 and Q_1 are linear over $(\mathbb{F}_2^w)^8$, each output word is the xor of three inputs;
- We identify $\mathbb{Z}_{2^w}^8$ and $(\mathbb{F}_2^w)^8$ with the natural mapping between them;
- We also define $\bar{\mu}(X^{[0]}, X^{[1]}, ...X^{[7]}) = \mu(X^{[7]}, X^{[6]}, ...X^{[0]})$.

Note that the quasi-group operation is very easy to invert: given X and $X*Y$, we can compute Y as $\nu^{-1}(X * Y - \mu(X))$.

The compression function takes as input 16 message ($M_{i,0}$ and $M_{i,1}$) words and 16 words of chaining value ($H_{i,0}$ and $H_{i,1}$) and produces 16 words of new chaining value ($H_{i+1,0}$ and $H_{i+1,1}$). The full compression function is described in Figure 1. For more details, see [4].

2.1 Previous analysis of EDON-$\mathcal{R}$

Previous work [5,6] has shown various weaknesses of the compression function:

- given $M_{i,0}$, $M_{i,1}$, $H_{i+1,0}$ and $H_{i+1,1}$, it is easy to compute $H_{i,0}$ and $H_{i,1}$;
- given $H_{i,0}$, $H_{i,1}$, $M_{i,0}$, and $H_{i+1,0}$, it is easy to compute $M_{i,1}$, and $H_{i+1,1}$;
- given $H_{i+1,1}$, $H_{i,0}$ and $M_{i,0}$, we can find a value of $H_{i,1}$, $H_{i+1,0}$, and $M_{i,1}$ with $2^{n/2}$ operations.

These results can be used to mount various attacks on the hash function:

- We can apply generic attacks against narrow-pipe hash functions: multi-collisions, second preimages on long message, fixed points, ...
- There is a preimage attack with complexity $2^{2n/3}$ and $2^{2n/3}$ memory.

The preimage attack requires less computations than a generic attack, but due to the large memory requirements, the machine to carry out this attack might be more expensive than a machine to perform a parallel brute force, so it is unclear whether this should be considered as an attack.

However, these results show that the compression function of EDON-$\mathcal{R}$ is quite weak, and the security of EDON-$\mathcal{R}$ cannot be based on a security proof of the Merkle-Damgård mode.

More recently, a work by Novotney and Ferguson [8] showed detectable biases in the output of the compression function.

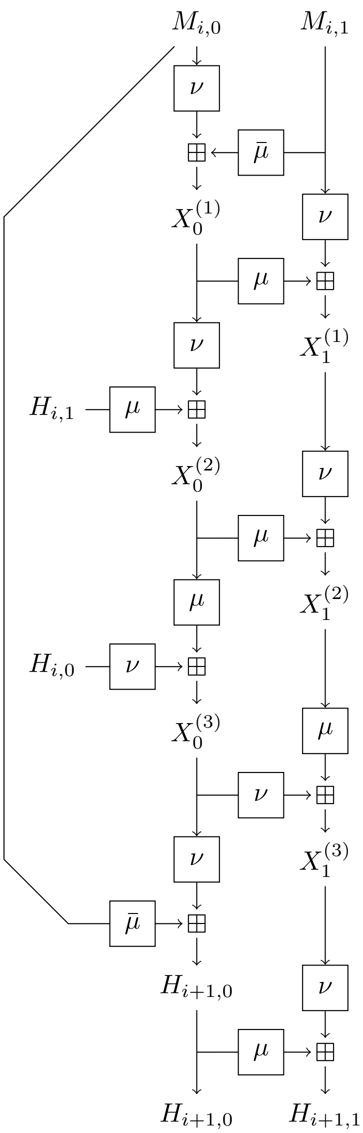

Fig. 1. EDON-$\mathcal{R}$ compression function.

2.2 Our Results

We present a new attack on the compression which allow to recover the full chaining value when half of the input chaining value and half of the output chaining value are known:

- given $M_{i,0}$, $M_{i,1}$, $H_{i,1}$ and $H_{i+1,1}$, we can compute $H_{i,0}$ and $H_{i+1,0}$.

In this paper we will describe two attacks: one that requires only two queries and a lot of computations, and a second with more queries and a practical complexity:

	Queries	Time	Memory	Precomputation
EDON-$\mathcal{R}$224/256	2	2^{160}	-	-
EDON-$\mathcal{R}$224/256	32	$\simeq 2^{30}$	-	2^{52}
EDON-$\mathcal{R}$384/512	2	2^{320}	-	-
EDON-$\mathcal{R}$384/512	32	$\simeq 2^{32}$	-	2^{100}

The attack on the compression function can be used to mount the following attacks on MAC constructions, with the same complexities as the attack on the compression function:

- A *key-recovery* attack against secret-IV EDON-$\mathcal{R}$;
- A *universal forgery* attack against secret-prefix EDON-$\mathcal{R}$ when the key is padded to full block;
- A *selective forgery* attack against secret-prefix EDON-$\mathcal{R}$ if the key is not padded to a full block (we can attack any message such that $k \| M$ takes more than one block after the padding);
- A *selective forgery* attack against the secret-prefix and secret-IV constructions when used with the tweaked EDON-$\mathcal{R}$ (we can attack any message that include a valid padding).

Our attacks only needs a few queries and negligible memory. They can easily be parallelized. Those attacks are the first attacks on the full EDON-$\mathcal{R}$ to clearly beat parallel generic attacks.

3 IV Recovery Using Related Queries

The first step of the IV-recovery attack is to gather information about the chaining values. We will make two calls to the MAC oracle, with two related messages, such that after the padding step, the first message is a prefix of the second one. The first message M is chosen arbitrarily such that after the padding it fits in one block pad(M). The second message M' has pad(M) as its first block, and has to fit in two blocks after the padding. This is similar to the length extension attack on narrow-pipe hash function. Applied to a wide-pipe design such as EDON-$\mathcal{R}$, this gives us some information on the input and output of the second compression function (see Fig 2):

- $M_{1,0}$ and $M_{1,1}$ are known;
- $H_{1,1}$ is known;
- $H_{2,1}$ is known.

We will show how to recover $H_{1,0}$. Then $H_{0,0}$ and $H_{0,1}$ can be recovered from $H_{1,0}, H_{1,1}$ and $M_{0,0}, M_{0,1}$ because the compression function of EDON-$\mathcal{R}$ is easy to invert [5]. Since there are 8 unknown words in the input of the compression

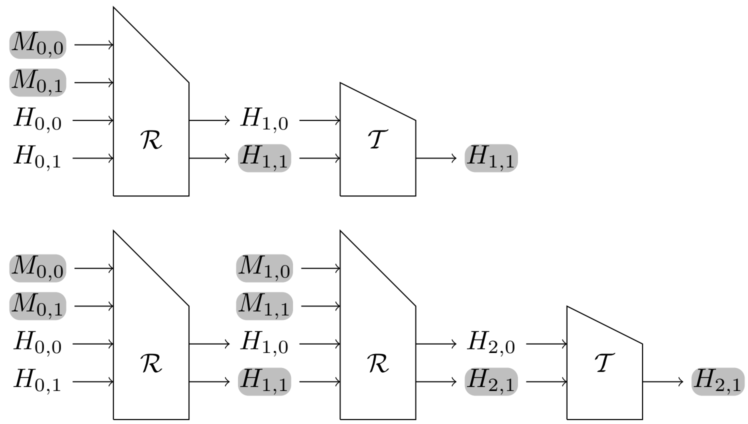

Fig. 2. The first message $\mathrm{pad}(M) = M_{0,0}M_{0,1}$ allow to recover $H_{1,1}$ while the second message $\mathrm{pad}(M') = M_{0,0}M_{0,1}M_{1,0}M_{1,1}$ allows to recover $H_{2,1}$.

function ($H_{1,0}$) and we know 8 words of the output of the compression function ($H_{2,1}$), we expect one solution on average. In this setting, a preimage attack will be able to recover *the* value of $H_{1,0}$ and not merely *a* value that gives the same output.

If we look at the description of the compression function [4], we have:

$$\begin{aligned} H_{2,1} &= H_{2,0} * X_1^{(3)} \\ &= (\overline{M_{1,0}} * X_0^{(3)}) * (X_1^{(2)} * X_0^{(3)}) \\ &= (\bar{\mu}(M_{1,0}) + \nu(X_0^{(3)})) * (\mu(X_1^{(2)}) + \nu(X_0^{(3)})) \\ &= (U + C_0) * (U + C_1) \end{aligned}$$

where $U = \nu(X_0^{(3)})$ is unknown, and $C_0 = \bar{\mu}(M_{1,0})$, $C_1 = \mu(X_1^{(2)})$ are known constants.

If we are able to solve the equation $H = (U + C_0) * (U + C_1)$ where U is the unknown, then we can recover $X_0^{(3)} = \nu^{-1}(U)$, and this will give us $H_{1,0} = \nu^{-1}(X_0^{(3)} - \mu(X_0^{(2)}))$.

4 Solving the equation $H = (U + C_0) * (U + C_1)$

The main step of the attack is to solve the equation

$$\begin{aligned} H &= (U + C_0) * (U + C_1) \\ &= Q_0(R_0(P_0(U + C_0))) + Q_1(R_1(P_1(U + C_1))) \end{aligned}$$

All the variables are 8-tuples of w bit words, and U is the unknown. To solve this equation, we will express U over a basis of $\mathbb{Z}_{2^w}^8$ such that some of the basis vectors do not affect some words of $(U+C_0)*(U+C_1)$. Then we can solve the equation more efficiently than by brute force because we do not need to explore the full space.

More precisely, P_0, P_1 are defined by the following matrices over $\mathbb{Z}_{2^w}$ (*i.e.*, the sums are modular additions):

$$P_0 = \begin{bmatrix} 1&1&1&0&1&0&0&1\\ 1&1&0&1&1&0&0&1\\ 1&1&0&0&1&0&1&1\\ 0&0&1&1&0&1&1&1\\ 0&1&1&1&0&1&1&0\\ 1&0&1&1&1&1&0&0\\ 1&1&0&0&0&1&1&1\\ 0&0&1&1&1&1&1&0 \end{bmatrix} \qquad P_1 = \begin{bmatrix} 1&1&1&0&0&1&0&1\\ 1&1&0&1&1&0&1&0\\ 1&1&1&1&0&1&0&0\\ 0&0&1&1&1&0&1&1\\ 1&1&0&1&1&1&0&0\\ 0&0&1&0&1&1&1&1\\ 0&1&1&0&0&1&1&1\\ 1&0&0&1&1&0&1&1 \end{bmatrix}$$

We will use three vectors U_0, U_1, U_2 in the kernels of some sub-matrices of P_0 and P_1:

$$\begin{aligned} U_0 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}^{\mathrm{T}} \\ U_1 &= \begin{bmatrix} 2 & 2 & 2 & 2 & 2^{31}-3 & 2^{31}-3 & 0 & 2^{31}-1 \end{bmatrix}^{\mathrm{T}} \\ U_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 2^{31}-1 & 2^{31} & 0 & 2^{31} \end{bmatrix}^{\mathrm{T}} \end{aligned}$$

Then we have (the question marks represent values for which we do not have any useful information):

$$P_0 \cdot U_0 = \begin{bmatrix} ? \; ? \; 0 \; 0 \; ? \; 0 \; 0 \; ? \end{bmatrix}^{\mathrm{T}} \qquad P_1 \cdot U_0 = \begin{bmatrix} ? \; ? \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \end{bmatrix}^{\mathrm{T}} \tag{1}$$

$$P_0 \cdot U_1 = \begin{bmatrix} ? \; ? \; 0 \; 0 \; ? \; 0 \; 0 \; ? \end{bmatrix}^{\mathrm{T}} \qquad P_1 \cdot U_1 = \begin{bmatrix} ? \; ? \; ? \; 0 \; 0 \; ? \; 0 \; 0 \end{bmatrix}^{\mathrm{T}} \tag{2}$$

$$P_0 \cdot U_2 = \begin{bmatrix} 0 \; 0 \; 0 \; 0 \; ? \; 0 \; ? \; ? \end{bmatrix}^{\mathrm{T}} \qquad P_1 \cdot U_2 = \begin{bmatrix} ? \; ? \; ? \; ? \; 0 \; ? \; 0 \; 0 \end{bmatrix}^{\mathrm{T}} \tag{3}$$

Q_0, Q_1 are defined by the following matrices over $\mathbb{F}_2^w$ (*i.e.*, the sums are exclusive or):

$$Q_0 = \begin{bmatrix} 1&1&0&0&1&0&0&0\\ 1&0&0&0&1&0&0&1\\ 0&1&0&0&0&0&1&1\\ 0&0&1&1&1&0&0&0\\ 1&1&0&0&0&0&0&1\\ 0&0&0&1&0&1&1&0\\ 0&0&1&0&0&1&1&0\\ 0&0&1&1&0&1&0&0 \end{bmatrix} \qquad Q_1 = \begin{bmatrix} 1&1&0&0&0&1&0&0\\ 0&0&1&0&0&0&1&1\\ 1&1&0&1&0&0&0&0\\ 1&0&0&1&1&0&0&0\\ 0&1&1&0&0&1&0&0\\ 0&0&0&1&1&0&1&0\\ 0&0&1&0&0&1&0&1\\ 0&0&0&0&1&0&1&1 \end{bmatrix}$$

Due to the positions of the zeros in $P_i \cdot U_j$, we have, for all $\alpha, \beta \in \mathbb{Z}_{2^w}$:

$$Q_0(R_0(P_0(X+\alpha U_0))) \oplus Q_0(R_0(P_0(X))) = \begin{bmatrix} ? \; ? \; ? \; ? \; ? \; 0 \; 0 \; 0 \end{bmatrix}^{\mathrm{T}} \tag{4}$$

$$Q_0(R_0(P_0(X+\alpha U_1))) \oplus Q_0(R_0(P_0(X))) = \begin{bmatrix} ? \; ? \; ? \; ? \; ? \; 0 \; 0 \; 0 \end{bmatrix}^{\mathrm{T}} \tag{5}$$

$$Q_0(R_0(P_0(X+\alpha U_2))) \oplus Q_0(R_0(P_0(X))) = \begin{bmatrix} ? \; ? \; ? \; ? \; ? \; ? \; ? \; 0 \end{bmatrix}^{\mathrm{T}} \tag{6}$$

$$Q_1(R_1(P_1(Y+\beta U_0))) \oplus Q_1(R_1(P_1(Y))) = \begin{bmatrix}? & ? & ? & ? & ? & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}} \tag{7}$$
$$Q_1(R_1(P_1(Y+\beta U_1))) \oplus Q_1(R_1(P_1(Y))) = \begin{bmatrix}? & ? & ? & ? & ? & 0 & ? & 0\end{bmatrix}^{\mathrm{T}} \tag{8}$$
$$Q_1(R_1(P_1(Y+\beta U_2))) \oplus Q_1(R_1(P_1(Y))) = \begin{bmatrix}? & ? & ? & ? & ? & ? & ? & 0\end{bmatrix}^{\mathrm{T}} \tag{9}$$

This proves that the vectors U_0, U_1, U_2 do not affect some of the output words. This property can be seen as a truncated differential for the $*$ operation:

$$(X+\alpha U_0) * (Y+\beta U_0) \oplus X * Y = \begin{bmatrix}? & ? & ? & ? & ? & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}} \tag{10}$$
$$(X+\alpha U_1) * (Y+\beta U_1) \oplus X * Y = \begin{bmatrix}? & ? & ? & ? & ? & 0 & ? & 0\end{bmatrix}^{\mathrm{T}} \tag{11}$$
$$(X+\alpha U_2) * (Y+\beta U_2) \oplus X * Y = \begin{bmatrix}? & ? & ? & ? & ? & ? & ? & 0\end{bmatrix}^{\mathrm{T}} \tag{12}$$

This is a very important part of the attack, so let us explain in more detail what equation (12) means. Using notations similar to the ones from [4], the last output word of $X * Y$ is computed as:

$$(X * Y)^{[7]} = (T_X^{[2]} \oplus T_X^{[3]} \oplus T_X^{[5]}) + (T_Y^{[4]} \oplus T_Y^{[6]} \oplus T_Y^{[7]})$$

where

$$\begin{aligned}
T_X^{[2]} &= (X^{[0]} + X^{[1]} + X^{[4]} + X^{[6]} + X^{[7]}) \lll 8 \\
T_X^{[3]} &= (X^{[2]} + X^{[3]} + X^{[5]} + X^{[6]} + X^{[7]}) \lll 13 \\
T_X^{[5]} &= (X^{[0]} + X^{[2]} + X^{[3]} + X^{[4]} + X^{[5]}) \lll 22 \\
T_Y^{[4]} &= (Y^{[0]} + Y^{[1]} + Y^{[3]} + Y^{[4]} + Y^{[5]}) \lll 15 \\
T_Y^{[6]} &= (Y^{[1]} + Y^{[2]} + Y^{[5]} + Y^{[6]} + Y^{[7]}) \lll 25 \\
T_Y^{[7]} &= (Y^{[0]} + Y^{[3]} + Y^{[4]} + Y^{[6]} + Y^{[7]}) \lll 27
\end{aligned}$$

We now consider $X' = X + \alpha U_2$ and $Y' = Y + \beta U_2$:

$$(X' * Y')^{[7]} = ({T'}_X^{[2]} \oplus {T'}_X^{[3]} \oplus {T'}_X^{[5]}) + ({T'}_Y^{[4]} \oplus {T'}_Y^{[6]} \oplus {T'}_Y^{[7]})$$

where

$$\begin{aligned}
{T'}_X^{[2]} &= (X^{[0]} + \alpha + X^{[1]} + X^{[4]} + \alpha(2^{31}-1) + X^{[6]} + X^{[7]} + \alpha 2^{31}) \lll 8 \\
{T'}_X^{[3]} &= (X^{[2]} + X^{[3]} + X^{[5]} + \alpha 2^{31} + X^{[6]} + X^{[7]} + \alpha 2^{31}) \lll 13 \\
{T'}_X^{[5]} &= (X^{[0]} + \alpha + X^{[2]} + X^{[3]} + X^{[4]} + \alpha(2^{31}-1) + X^{[5]} + \alpha 2^{31}) \lll 22 \\
{T'}_Y^{[4]} &= (Y^{[0]} + \beta + Y^{[1]} + Y^{[3]} + Y^{[4]} + \beta(2^{31}-1) + Y^{[5]} + \beta 2^{31}) \lll 15 \\
{T'}_Y^{[6]} &= (Y^{[1]} + Y^{[2]} + Y^{[5]} + \beta 2^{31} + Y^{[6]} + Y^{[7]} + \beta 2^{31}) \lll 25 \\
{T'}_Y^{[7]} &= (Y^{[0]} + \beta + Y^{[3]} + Y^{[4]} + \beta(2^{31}-1) + Y^{[6]} + Y^{[7]} + \beta 2^{31}) \lll 27
\end{aligned}$$

We see that the α and β terms cancels out:

$$\begin{array}{ccc} T_X^{[2]} = T'^{[2]}_X & T_X^{[3]} = T'^{[3]}_X & T_X^{[5]} = T'^{[5]}_X \\ T_Y^{[4]} = T'^{[4]}_Y & T_Y^{[6]} = T'^{[6]}_Y & T_Y^{[7]} = T'^{[7]}_Y \end{array}$$

and as a consequence $(X' * Y')^{[7]} = (X * Y)^{[7]}$. This works because U_2 was chosen in the kernel of the linear forms that define $T_X^{[2]}$, $T_X^{[3]}$, $T_X^{[5]}$, $T_Y^{[4]}$, $T_Y^{[6]}$, and $T_Y^{[7]}$. Similarly, U_1 is in the kernel of the linear forms involved in the computation of $(X*Y)^{[5,7]}$ and U_0 is in the kernel of the linear forms involved in the computation of $(X * Y)^{[5,6,7]}$.

Thanks to this property, we can do an exhaustive search with early abort. We extend U_0, U_1, U_2 into a basis $U_0, U_1, ...U_7$[1] of $\mathbb{Z}_{2^w}^8$, and we will represent U in this basis: $U = \sum_{i=0}^{7} \alpha_i U_i$. We define $V = (U + C_0) * (U + C_1)$. Due to the properties of U_0, U_1, U_2, we know that:

- α_0 has no effect on $V^{[5]}$, $V^{[6]}$ and $V^{[7]}$;
- α_1 has no effect on $V^{[5]}$ and $V^{[7]}$;
- α_2 has no effect on $V^{[7]}$.

The full algorithm is given by Algorithm 1 and is quite simple. We first iterate over $\alpha_3, \alpha_4, ...\alpha_7$ and we filter the elements such that $V = (U + C_0) * (U + C_1)$ matches H on the last coordinates. If it does not match, we do not need to iterate over $\alpha_0, \alpha_1, \alpha_2$ because this will not modify $V^{[7]}$, so we can abort this branch. For the choices that match, we iterate over α_2 and check $V^{[5]}$. If it matches $H^{[5]}$, we iterate over α_1 and check $V^{[6]}$. If it matches $H^{[6]}$, we can then iterate over α_0.

The time complexity is $2^{5w} = 2^{5n/8}$:

- the first loop is executed 2^{5w} times;
- each matching reduces the number of candidates to 2^{4w};
- each subsequent loop raises the number of candidates to 2^{5w}.

The memory requirements are negligible because we do not need to store a list of candidate. We just perform a breath-first search and we prune the bad branches to reduce the size of the tree.

Once we have recovered $U = \nu(X_0^{(3)})$, it is easy to invert the permutations and recover $X_0^{(3)}$. From that we find $H_{1,0}$ by inverting a quasi-group operation, and we have all the variables of the compression function. We can then recover the key $H_{0,0}, H_{0,1}$ by inverting the first compression function (it is easy when the output and the message are known).

[1] For instance, we can use:

$$\begin{array}{lll} U_3 = [0,0,1,0,0,0,0,0]^T & U_5 = [0,0,0,0,1,0,0,0]^T & U_7 = [0,0,0,0,0,0,0,1]^T \\ U_4 = [0,0,0,1,0,0,0,0]^T & U_6 = [0,0,0,0,0,1,0,0]^T & \end{array}$$

Algorithm 1 Solving $H = (U + C_0) * (U + C_1)$

Input: C_0, C_1, H
Output: U

for all $\alpha_3, \alpha_4, ...\alpha_7 \in \mathbb{Z}_{2^w}$ **do**
  $U \leftarrow \sum_{i=3}^{7} \alpha_i U_i$
  $V \leftarrow (U + C_0) * (U + C_1)$
  if $V^{[7]} = H^{[7]}$ **then**
    for all $\alpha_2 \in \mathbb{Z}_{2^w}$ **do**
      $U \leftarrow \sum_{i=2}^{7} \alpha_i U_i$
      $V \leftarrow (U + C_0) * (U + C_1)$
      if $V^{[5]} = H^{[5]}$ **then**
        for all $\alpha_1 \in \mathbb{Z}_{2^w}$ **do**
          $U \leftarrow \sum_{i=1}^{7} \alpha_i U_i$
          $V \leftarrow (U + C_0) * (U + C_1)$
          if $V^{[6]} = H^{[6]}$ **then**
            for all $\alpha_0 \in \mathbb{Z}_{2^w}$ **do**
              $U \leftarrow \sum_{i=0}^{7} \alpha_i U_i$
              $V \leftarrow (U + C_0) * (U + C_1)$
              if $V = H$ **then**
                U is a solution

5 Using more queries

In this section, we improve this attack using more queries to the MAC oracle. We gather more equations of the form $H = (U + C_0) * (U + C_1)$, and this enables us to mount a very efficient attack. In the case of Edon-$\mathcal{R}$256, it requires about 32 queries and can recover the secret key with about 2^{30} computations after a precomputation of about 2^{52} operations, which makes it a practical attack.

5.1 Building the queries

To get new equations, we will query the MAC oracle with new messages $M^{(i)}$ so that pad(M) is a prefix of all the $M^{(i)}$'s. Each query will give some equation involving the same $H_{1,0}$, and we will deduce an equation of the form $H^{(i)} = (U + C_0^{(i)}) * (U + C_1^{(i)})$ as in the previous section. Remember that we have $U = \nu(X_0^{(3)}) = \nu(\nu(H_{1,0}) + \mu(X_0^{(2)}))$. We will build our messages so that the value of $X_0^{(2)}$ is the same for all the $M^{(i)}$'s, or equivalently, $X_0^{(1)}$ is the same for all the $M^{(i)}$'s. This means that all the equations will involve the same U, and recovering this U will allow to recover $H_{1,0}$.

Let us assume that we have two such equations, and let us further assume that $C_0^{(i)} = C_0^{(j)}$. Then:

$$\begin{aligned}
H^{(i)} &= Q_0(R_0(P_0(U + C_0^{(i)}))) + Q_1(R_1(P_1(U + C_1^{(i)}))) \\
H^{(j)} &= Q_0(R_0(P_0(U + C_0^{(j)}))) + Q_1(R_1(P_1(U + C_1^{(j)}))) \\
H^{(i)} - H^{(j)} &= Q_1(R_1(P_1(U + C_1^{(i)}))) - Q_1(R_1(P_1(U + C_1^{(j)})))
\end{aligned}$$

since P_1 is linear over $\mathbb{Z}_{2^w}^8$, we can consider $\tilde{U} = P_1 \cdot U$ and $\tilde{C}_1^{(i)} = P_1 \cdot C_1^{(i)}$

$$H^{(i,j)} = H^{(i)} - H^{(j)} = Q_1(R_1(\tilde{U} + \tilde{C}_1^{(i)})) - Q_1(R_1(\tilde{U} + \tilde{C}_1^{(j)})) \tag{13}$$

If we consider $\tilde{U} = P_1 \cdot U$ to be the unknown, this gives a simpler equation than in the previous section, where $H^{(i,j)}$, $\tilde{C}_1^{(i)}$ and $\tilde{C}_1^{(j)}$ are known constants.

However, if we have a pair of messages $M^{(i)}, M^{(j)}$ where $C_0^{(i)} = C_0^{(j)}$ and $X_0^{(1)}$ is constant, then we have $M^{(i)} = M^{(j)}$ and we can only build a trivial equation. Instead, we use messages such that only some words of $C_0^{(i)}$ and $C_0^{(j)}$ are equal. Namely, if we have

$$(P_0 \cdot C_0^{(i)})^{[2,3,5]} = (P_0 \cdot C_0^{(j)})^{[2,3,5]} \tag{14}$$

then

$$\begin{aligned}
H^{(i,j)[7]} &= \Big(\mu(U + C_0^{(i)}) + \nu(U + C_1^{(i)})\Big)^{[7]} - \Big(\mu(U + C_0^{(j)}) + \nu(U + C_1^{(j)})\Big)^{[7]} \\
&= \Big(\nu(U + C_1^{(i)}) - \nu(U + C_1^{(j)})\Big)^{[7]} + \Big(\mu(U + C_0^{(i)}) - \mu(U + C_0^{(j)})\Big)^{[7]} \\
&= \Big(\nu(U + C_1^{(i)}) - \nu(U + C_1^{(j)})\Big)^{[7]} \\
&\quad + \Big(P_0(U + C_0^{(i)})^{[2]} \ggg 8 \oplus P_0(U + C_0^{(i)})^{[3]} \ggg 13 \oplus P_0(U + C_0^{(i)})^{[5]} \ggg 22\Big) \\
&\quad - \Big(P_0(U + C_0^{(j)})^{[2]} \ggg 8 \oplus P_0(U + C_0^{(j)})^{[3]} \ggg 13 \oplus P_0(U + C_0^{(j)})^{[5]} \ggg 22\Big) \\
H^{(i,j)[7]} &= Q_1(R_1(\tilde{U} + \tilde{C}_1^{(i)}))^{[7]} - Q_1(R_1(\tilde{U} + \tilde{C}_1^{(j)}))^{[7]}
\end{aligned} \tag{15}$$

The two gray terms cancel out by linearity of P_0 over $\mathbb{Z}_{2^w}^8$. We can see (15) as a weaker version of (13): we only have an equation on one word, instead of eight. We can build similar equations restricted to any word by choosing appropriate relations between $C_0^{(i)}$ and $C_0^{(j)}$: if we want an equation restricted to word k we just need to have an equality between $P_0 \cdot C_0^{(i)}$ and $P_0 \cdot C_0^{(j)}$ on the three words used in the computation of $Q_0^{[k]}$.

5.2 Dealing with the padding

Another problem that we face to gather these equations is the padding. EDON-$\mathcal{R}$ uses a padding with Merkle-Damgård strengthening, so there are 65 bits in $M_{1,1}$ that must be kept untouched (129 bits in EDON-$\mathcal{R}$384/512).

To find proper messages, we use a preprocessing step. First, we fix some arbitrary value for $X_0^{(1)}$. Then we take a set of random $M_{1,1}$ satisfying the padding, we compute the corresponding $M_{1,0}$ and we look for a collision in three words of $P_0 \cdot C_0^{(i)}$ according to (14). Each collision costs 2^{48} computations on average (2^{96} for EDON-$\mathcal{R}$384/512), and gives one equation. Note that this is independent of the key we are attacking. It can be done as a preprocessing step, and we only need to store a the message pairs that will be used to extract the equations. Since we need 16 collisions, the time complexity of this preprocessing step will be 16×2^{48} for EDON-$\mathcal{R}$256 and 16×2^{96} for EDON-$\mathcal{R}$512.

5.3 Solving

To recover the value of U, we gather several equation of the type of (15). We can rewrite them as:

$$\Big((\tilde{U}^{[4]} + \tilde{C}_1^{(i)[4]}) \ggg 17 \oplus (\tilde{U}^{[6]} + \tilde{C}_1^{(i)[6]}) \ggg 7 \oplus (\tilde{U}^{[7]} + \tilde{C}_1^{(i)[7]}) \ggg 5\Big) - \tag{16}$$
$$\Big((\tilde{U}^{[4]} + \tilde{C}_1^{(j)[4]}) \ggg 17 \oplus (\tilde{U}^{[6]} + \tilde{C}_1^{(j)[6]}) \ggg 7 \oplus (\tilde{U}^{[7]} + \tilde{C}_1^{(j)[7]}) \ggg 5\Big) = H^{(i,j)[7]}$$

We will solve these equations using a guess-and-determine approach. First we guess the 18 lower bits of $\tilde{U}^{[4]}$, the 8 lower bits of $\tilde{U}^{[6]}$, and the 6 lower bits of $\tilde{U}^{[7]}$. This allows us to compute the least significant bit of the left hand side of (16), and we check this bit against the right hand side. If we have enough equations, we can filter out many bad candidates. Then we guess one more bit of $\tilde{U}^{[4]}$, $\tilde{U}^{[6]}$, and $\tilde{U}^{[7]}$. We can now compute one more bit of (16), and again reduce the number of candidates. We repeat this step until all the bits of $\tilde{U}^{[4]}$, $\tilde{U}^{[6]}$, and $\tilde{U}^{[7]}$ have been guessed. Each time we guess some bits, the number of candidates grows, but it will shrink when we check the new bit of (16). The cost of this step is at least 2^{32} because we have to guess 32 bits in the beginning. If we have enough equations and they give an independent filtering, we expect the complexity to be about 2^{32}. We did some experiments with random constants to check our assumptions. Experiments shows that with only 10 equations we can solve (16) for EDON-$\mathcal{R}$256 by exploring slightly more than 2^{32} nodes. This take a few minutes on a desktop PC. For EDON-$\mathcal{R}$512, we have to guess 56 bits, and we expect a complexity of 2^{56}.

Another way to solve this system is to guess the carries instead of guessing the low order bits. In this case, we only use 4 equations, because we have to guess the carries in each equations. We have only 24 carry bits to guess, but the 4 equations have many solutions, so we use extra equations to check each of these solutions until a single solution is left. According to our experiments, this takes about one minute on a desktop PC, and we have about 2^{16} solutions when using 4 equations (the search goes through 2^{30} nodes). Note that the complexity of this technique is independent of the rotation amounts, so it can be applied with any output word, not necessarily the seventh as in (15). More importantly, it is about as efficient on EDON-$\mathcal{R}$512: it take about 20 minutes to explore 2^{33} nodes, and gives about 2^{20} solutions.

This first step gives us $\tilde{U}^{[4]}$, $\tilde{U}^{[6]}$, and $\tilde{U}^{[7]}$. Next, we use an equation similar to (15), but involving the fifth word instead of the seventh:

$$\Big((\tilde{U}^{[3]} + \tilde{C}_1^{(i)[3]}) \ggg 21 \oplus (\tilde{U}^{[4]} + \tilde{C}_1^{(i)[4]}) \ggg 17 \oplus (\tilde{U}^{[6]} + \tilde{C}_1^{(i)[6]}) \ggg 7\Big) - \tag{17}$$
$$\Big((\tilde{U}^{[3]} + \tilde{C}_1^{(j)[3]}) \ggg 21 \oplus (\tilde{U}^{[4]} + \tilde{C}_1^{(j)[4]}) \ggg 17 \oplus (\tilde{U}^{[6]} + \tilde{C}_1^{(j)[6]}) \ggg 7\Big) = H^{(i,j)[5]}$$

Since this equation only involves one unknown word $\tilde{U}^{[3]}$, it is quite easy to solve. We use the same technique as previously: we guess the carry bits. We only have 2 carry bits to guess so this step is negligible. We will repeat this using different equations involving different words of $\tilde{U}$, so as to recover the words of $\tilde{U}$ one by one. Then, we can recover $U = P_1^{-1} \cdot \tilde{U}$, and finally $H_{1,0}$.

The number of queries needed for the attack is 30: 2×10 to recover three words in the first step and 2 for each subsequent word.

Conclusion

We have shown a practical key-recovery attack against secret-IV EDON-$\mathcal{R}$ and various forgery attacks on secret-prefix EDON-$\mathcal{R}$. Moreover, we show that a selective forgery attack can still be done against the tweaked EDON-$\mathcal{R}$. While those constructions are not required to be secure by NIST, it is a natural construction that is used in some protocols. We believe that a strong cryptographic hash function should not leak the key when used in this setting.

Acknowledgement

Part of this work is supported by the Commission of the European Communities through the IST program under contract IST-2002-507932 ECRYPT, by the French government through the Saphir RNRT project, and by the French DGA.

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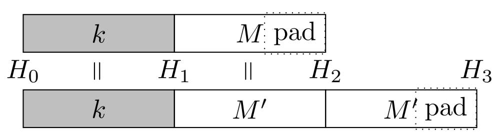
A Extension to other settings

The IV-recovery attack can be use to break the secret-prefix construction when used with Edon-$\mathcal{R}$, and the secret-IV and secret-prefix construction when used with the tweaked version of Edon-$\mathcal{R}$. If this section we describe the attacks with only two queries, but the attacks with 32 queries can be adapted in the same way.

A.1 Secret-prefix Edon-$\mathcal{R}$

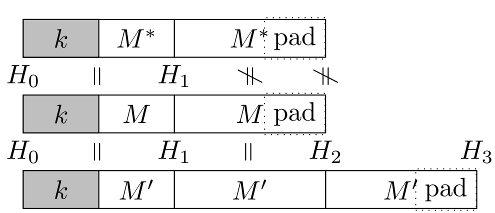
If the key is padded to a full block, we can recover the chaining value H_1 after processing the key. This chaining value will allow an attacker to compute the MAC of any message:



pad($k \| M$) is a prefix of $k \| M'$.

We apply the attack on the third compression function. We can recover H_2, and compute H_1 by inverting the second compression function.

If the key is not padded to a full block, we have a selective forgery attack. Given a message M^* such that pad($k \| M^*$) has at least two blocks, we use a message M such that the first block of pad($k \| M$) is equal to the first block of pad($k \| M^*$).



The first block is the same.

pad($k \| M$) is a prefix of $k \| M'$.

We apply the attack on the third compression function. Again, we can recover H_2, and compute H_1 by inverting the second compression function. Then, we can forge the MAC of M^*.

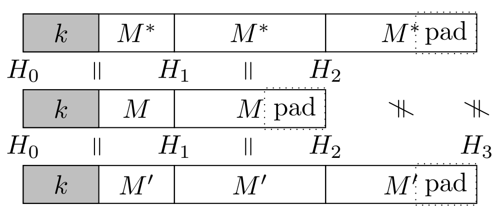
A.2 The tweaked version of Edon-$\mathcal{R}$

In [3], Gligoroski and Klima proposed a tweak to address the attacks found against Edon-$\mathcal{R}$. The tweak is described as:

> Instead of the old compression function $\mathcal{R}(oldPipe, M)$, now the compression function have the following feedback: $\mathcal{R}(oldPipe, M) \oplus oldPipe \oplus M'$, where M is represented in two parts i.e. $M = (M_0, M_1)$, and $M' = (M_1, M_0)$.

It is easy to see that our attack on the compression function can still be applied: if we know the right half of *oldPipe*, the right half of the output of the compression function, and the message block, we can compute the right half of $\mathcal{R}(oldPipe, M)$ from the right half of $\mathcal{R}(oldPipe, M) \oplus oldPipe \oplus M'$. However, we can no longer invert the compression function. Therefore, in the IV-recovery attack from Section 3, we can recover $(H_{1,0}, H_{1,1})$ using the attack on the compression function, but we can not recover the key $(H_{0,0}, H_{0,0})$.

Still, we have a selective forgery attack on the secret-prefix and secret-IV constructions. Let us describe the attack on the secret-prefix construction. The forgery will work for messages M^* such that some prefix of $k\|M^*$ is a valid padded message. For instance, we can fix 65 bits (129 in the case of Edon-$\mathcal{R}$512) at the end of the second block. A random message of 2^ℓ blocks can be attacked with probability $2^{\ell-65}$ ($2^{\ell-129}$ for Edon-$\mathcal{R}$512). Given such a message, we use a message M such that pad$(k\|M)$ is a prefix of $k\|M^*$:



pad$(k\|M)$ is a prefix of $k\|M^*$.

pad$(k\|M)$ is a prefix of $k\|M'$.

We apply the attack on the following block and we recover the inner state after processing pad$(k\|M)$. Since pad$(k\|M)$ is a prefix of M^*, we can use this information to forge the MAC of M^*.