Practical Key Recovery Attack against Secret-IV EDON-$\mathcal{R}$

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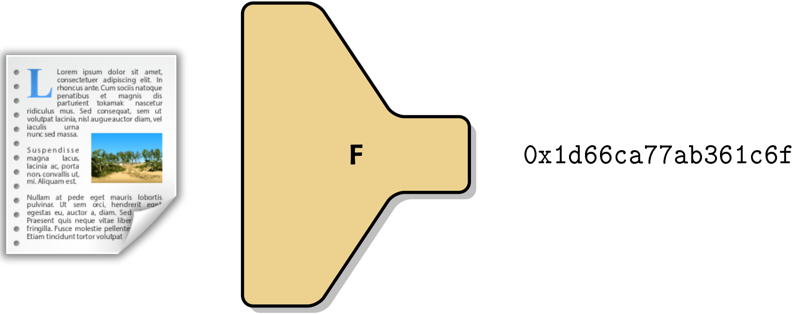
What is a hash function?

- A public function with no structural properties.
  - Cryptographic strength without keys!
  - We also expect security when used in a keyed mode...

- $F : \{0,1\}^* \to \{0,1\}^n$



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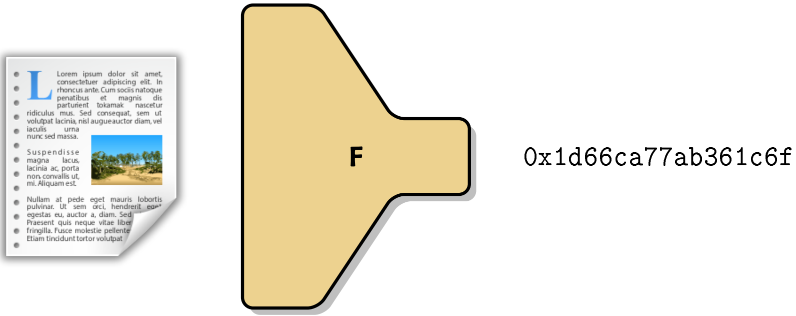

The SHA-3 Competition

- Similar to the AES competition
- Organized by NIST
- After devastating attacks on MD4, MD5, SHA-1, ...

- Submission dead-line was October 2008: 64 candidates
- 51 valid submissions in first round (December 2008)

- 14 in the second round (July 2009)
- 5 finalists in September 2010?
- 1 winner in 2012?

New designs

- Take into consideration recent advances in cryptanalysis
- Somewhat higher expectation that SHA-2
- Wide diversity of designs
- EDON-$\mathcal{R}$ was one of the first round candidates
  - One of the fastest.
- In this work, we study EDON-$\mathcal{R}$ used as a MAC

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Submission to the NIST SHA-3 competition

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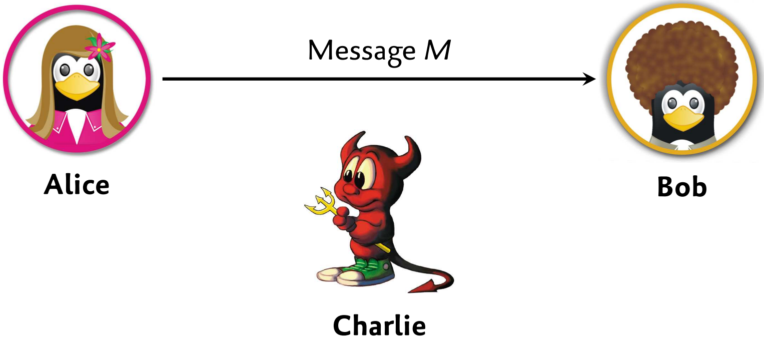

- Alice wants to send a message to Bob
- They share a key, and use a MAC to authenticate the message
- Bob rejects the message if $\text{MAC}_k(M) \neq t$

What is a MAC algorithm?

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MAC security

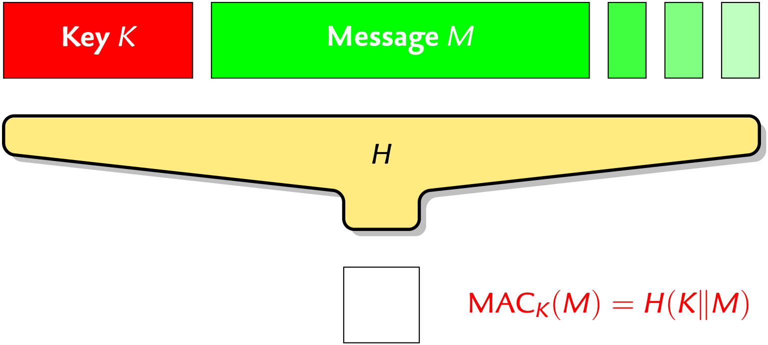
A MAC (Message Authentication Code) should provide authentication and integrity protection.

MAC security notions: chosen message attacks

The adversary has access to an oracle $M \mapsto \mathrm{MAC}_k(M)$.
He must compute a new MAC for:

- One message of his choice: existential forgery.
- A challenge message: selective forgery.
- Any message: universal forgery.

Secret-prefix MAC



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- The hash function is expected to break the structure of the input
  - This is the secret-prefix construction
- MD5, SHA-1 cannot be used in this way...
  - But new designs *should* try to be safe in this mode

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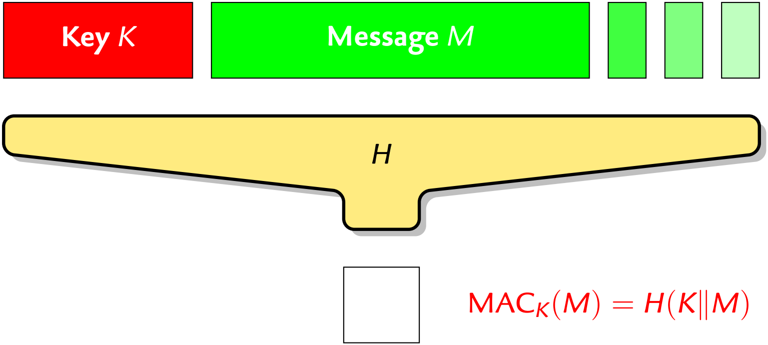

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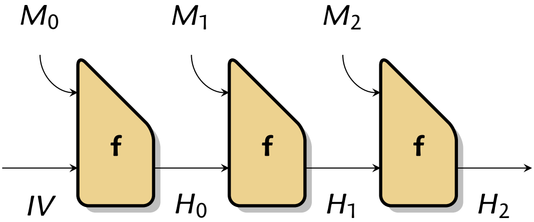
Overview of the attack

We want to extract key information from the state.

1. Use related queries to gather information about input and output of the round function $\mathcal{R}$
   - The compression function can be reduced to a small equation
2. Solve using linear algebra techniques
   - Using only two queries
   - Or using more queries to decrease the time complexity

Length extension attack

- Most hash function are based on a simple iteration.



- In secret-prefix MAC, we don't know the inner state
- But given the MAC of a known message, we can compute the MAC of some related messages (existential forgery).
- Solution: use a finalisation function.

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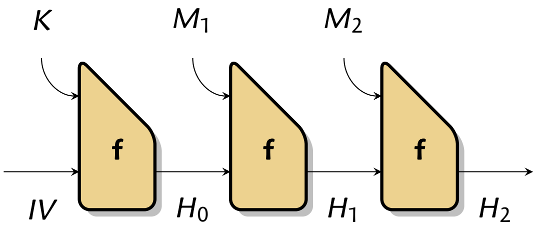

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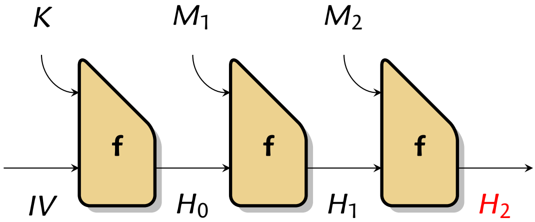

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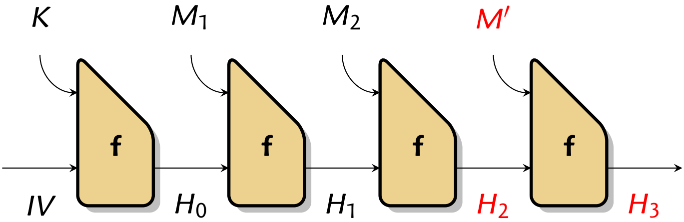

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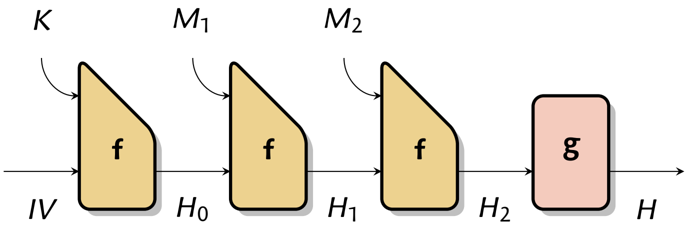

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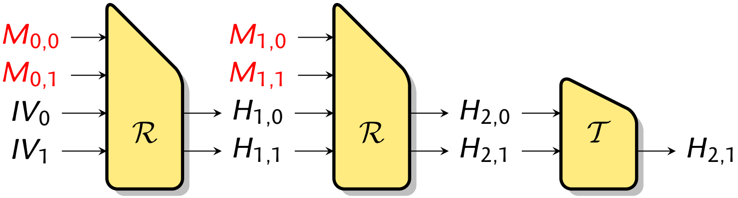
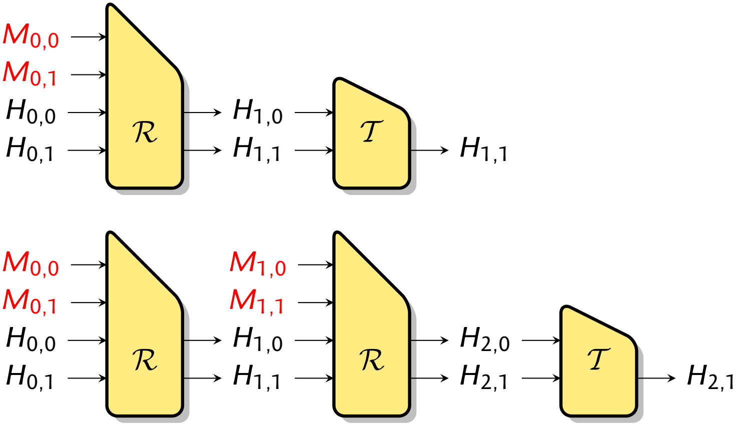
EDON-$\mathcal{R}$ *iteration*

- EDON-$\mathcal{R}$ is based on the chop-MD principle
  - The internal state is twice as big as the output
  - The finalization function is just a truncation.



- Can we do something similar to the length-extension attack?

Using Related Queries



- We use two related messages: M is a prefix of M'
- $\mathrm{MAC}(M)$ gives $H_{1,1}$
- $\mathrm{MAC}(M')$ gives $H_{2,1}$

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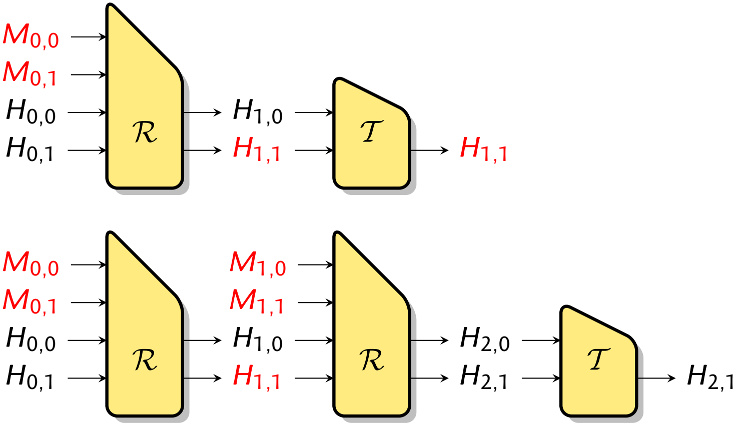

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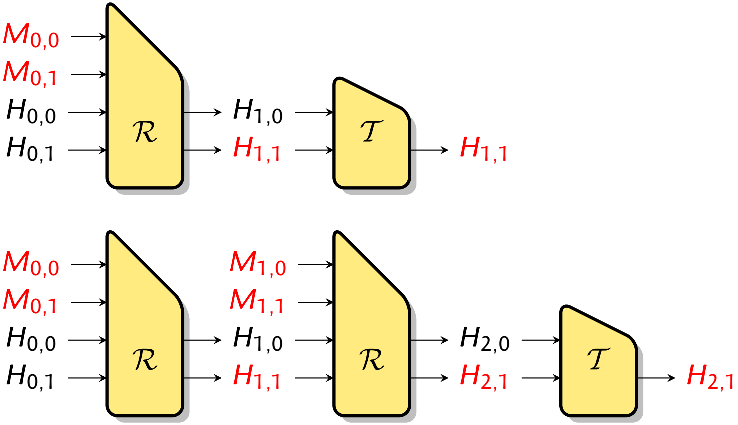

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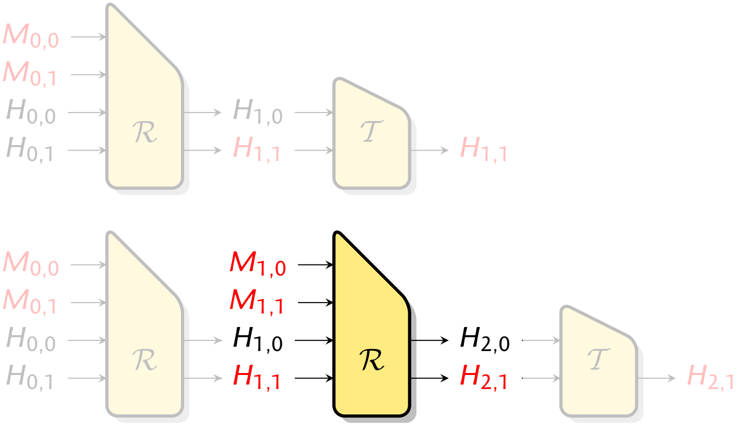

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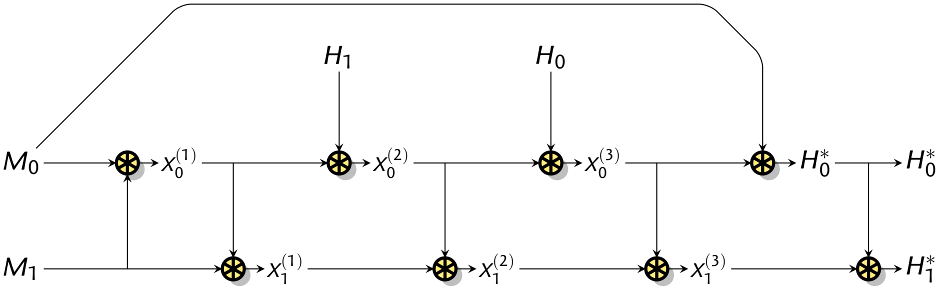
EDON-$\mathcal{R}$ Compression function



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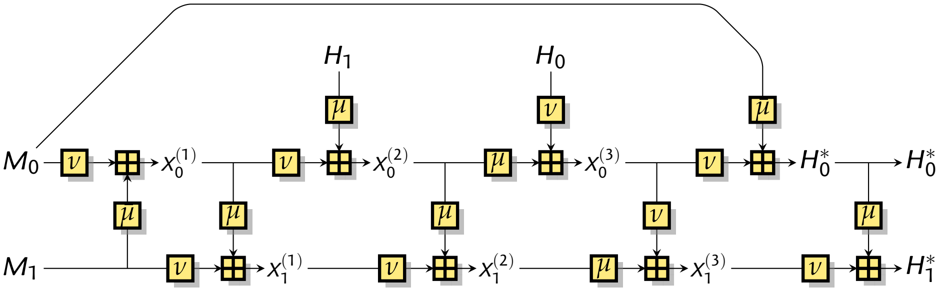
- $*$ is a sum of two permutations, μ and ν

EDON-$\mathcal{R}$ Compression function



- Design based on a quasi-group operation $*$
- $*$ is a sum of two permutations, μ and ν

The Quasi-group Operation $*$

$$X * Y = \mu(X) + \nu(Y)$$

Inside μ

1. A linear step over $\mathbb{Z}_{2^w}^8$ $\qquad S_X = P_X(X)$

   $S_X^{[2]} = X^{[0]} + X^{[1]} + X^{[4]} + X^{[6]} + X^{[7]}$

2. Word-wise rotations $\qquad T_X = R_X(S_X)$

   $T_X^{[2]} = S_X^{[2]} \lll 8$

3. A linear step over $(\mathbb{F}_2^w)^8$ $\qquad \mu(X) = Q_X(T_X)$

   $\mu(X)^{[7]} = T_X^{[2]} \oplus T_X^{[3]} \oplus T_X^{[5]}$

The Quasi-group Operation $*$

$$X * Y = \mu(X) + \nu(Y)$$

Inside ν

1. A linear step over $\mathbb{Z}_{2^w}^8$ — $S_Y = P_Y(Y)$

   $S_Y^{[0]} = Y^{[0]} + Y^{[1]} + Y^{[3]} + Y^{[4]} + Y^{[5]}$

2. Word-wise rotations — $T_Y = R_Y(S_Y)$

   $T_Y^{[4]} = S_Y^{[4]} \lll 15$

3. A linear step over $(\mathbb{F}_2^w)^8$ — $\nu(Y) = Q_Y(T_Y)$

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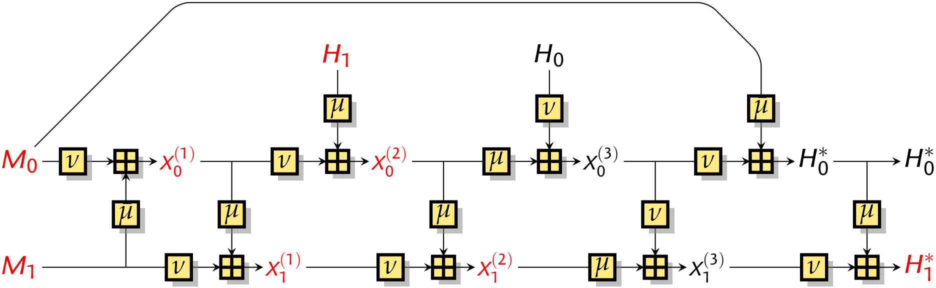
Simplify



- We know M_0, M_1, H_1 and H_1^*
  - We can compute some parts
- $C_0 = \bar{\mu}(M_{1,0})$, $C_1 = \mu(X_1^{(2)})$, $C_2 = \mu(X_0^{(2)})$
- $U = \nu(X_0^{(3)})$ is the new unknown $\quad (H_0 = \nu^{-1}(\nu^{-1}(U) - C_2))$
- $H_1^* = (U + C_0) * (U + C_1)$
  - Relatively simple equation

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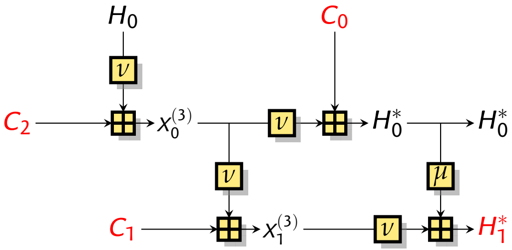

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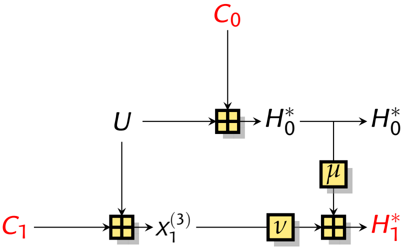

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  - Relatively simple equation

$H_1^* = (U + C_0) * (U + C_1)$

The core of the attack is to solve this equation:

$$H_1^* = (U + C_0) * (U + C_1)$$

We propose two techniques:

1. A technique based on neutral words
2. We build an even simpler equation using several equations

Neutral words

- We identified a subspace of the input that does not affect the full output of $*$

$$
\begin{array}{llllllllll}
U_0 = [& 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 &] \\
U_1 = [& 2 & 2 & 2 & 2 & 2^{31}-3 & 2^{31}-3 & 0 & 2^{31}-1 &] \\
U_2 = [& 1 & 0 & 0 & 0 & 2^{31}-1 & 2^{31} & 0 & 2^{31} &]
\end{array}
$$

$$
\begin{aligned}
(X+\alpha U_0) * (Y+\beta U_0) \oplus X * Y &= [* \;\; * \;\; * \;\; * \;\; * \;\; 0 \;\; 0 \;\; 0] \\
(X+\alpha U_1) * (Y+\beta U_1) \oplus X * Y &= [* \;\; * \;\; * \;\; * \;\; * \;\; 0 \;\; * \;\; 0] \\
(X+\alpha U_2) * (Y+\beta U_2) \oplus X * Y &= [* \;\; * \;\; * \;\; * \;\; * \;\; * \;\; * \;\; 0]
\end{aligned}
$$

Using neutral words

1. Extend U_0, U_1, U_2 into a basis $U_0, \ldots, U_7$
   - Write the unknown in this basis $U = \sum_{i=0}^{7} \alpha_i U_i$

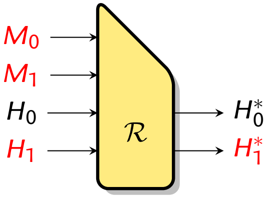
2. For each choice of $\alpha_3, \ldots \alpha_7$
   We know that $\alpha_0, \alpha_1, \alpha_2$ do not affect $H^{[7]}$
   Compute $H^{[7]}$ and skip if it is wrong.

3. Similarly, we can filter on $H^{[5]}$ after choosing α_2

4. And filter on $H^{[6]}$ after choosing α_1

Our results: Attack I

- Attack on the compression function:
  - Given M_0, M_1, H_1 and H_1^*, we can compute H_0 and H_0^*.



- We use two MAC queries to get H_1 and H_1^*.

	Queries	Time	Memory	Precomputation
EDON-$\mathcal{R}$224/256	2	2^{160}	-	-
EDON-$\mathcal{R}$384/512	2	2^{320}	-	-

Using more queries

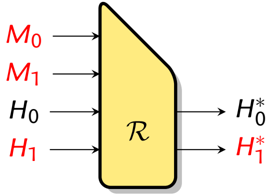
- We build two equations with *similar* C_0 constants
- We subtract the two equations
  - The μ terms *mostly* cancels out

$$
\begin{aligned}
H^{(i)} &= \mu(U + C_0^{(i)}) + \nu(U + C_1^{(i)}) \\
H^{(j)} &= \mu(U + C_0^{(j)}) + \nu(U + C_1^{(j)}) \\
H^{(i)} - H^{(j)} &\approx \nu(U + C_1^{(i)}) - \nu(U + C_1^{(j)})
\end{aligned}
$$

- We can remove the first layer of ν by linearity

Our results: Attack II

- Attack on the compression function:
  - Given M_0, M_1, H_1 and H_1^*, with a specially crafted message pair, we can build a simple equation involving H_0 and H_0^*.
  - With 10 related equations, we can recover three words of H_0.



- We need two MAC queries for each equation.

	Queries	Time	Memory	Precomputation
EDON-$\mathcal{R}$224/256	32	$\simeq 2^{30}$	-	2^{52}
EDON-$\mathcal{R}$384/512	32	$\simeq 2^{32}$	-	2^{100}

Thank you for listening!

Any questions?

Neutral words: example

$$X' = X + \alpha U_2, Y' = Y + \beta U_2$$
$$(X * Y)^{[7]} = (T_X^{[2]} \oplus T_X^{[3]} \oplus T_X^{[5]}) + (T_Y^{[4]} \oplus T_Y^{[6]} \oplus T_Y^{[7]})$$

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