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Automatic Search of Differential Path in MD4

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Motivation

Why do we need an algorithm?

- Understanding
- Improving
- New attacks

Results

- Some improvement of known attacks
- New attack against NMAC-MD4

Outline

1. Introduction
   - The MD4 hash function
   - Wang's attack
2. Understand and automate
   - Sufficient conditions
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   - Differential Path
   - Message difference
3. Results
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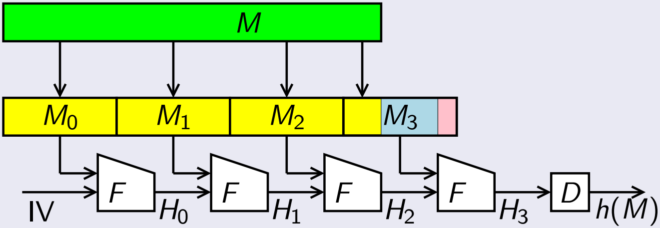
4. Conclusion

The MD4 hash function

General design

MD4 Design

- Merkle-Damgård
- Block size: 512 bits
- Internal state: 128 bits
- MD Strengthening



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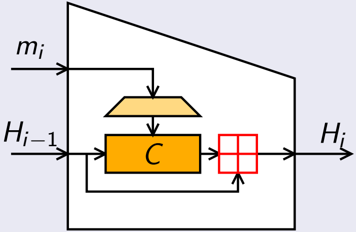
The MD4 hash function

Compression function

Compression Function Design

- Davies-Meyer with a Feistel-like cipher.

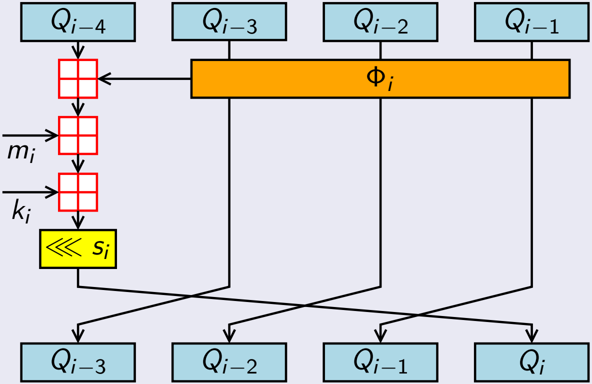


- Designed to be fast: 32 bit words, and operations available in hardware:
  - additions mod2^{32}: $\boxplus$
  - boolean functions: Φ_i
  - rotations $\lll s_i$
- Message expansion $M = \langle M_0, \ldots M_{15} \rangle \mapsto \langle m_0, \ldots m_{47} \rangle$
- 4 words of internal state Q_i updated in rounds of 16 steps

The MD4 hash function

Compression function

MD4 Step Update



$$Q_i = (Q_{i-4} \boxplus \Phi_i(Q_{i-1}, Q_{i-2}, Q_{i-3}) \boxplus m_i \boxplus k_i) \lll s_i$$

MD4 Collisions

Wang in a nutshell

1. Precomputation:
   - Choose a message difference.
   - Compute a differential path.
   - Derive a set of sufficient conditions.
2. Collision search:
   - Find a message that satisfies the set of conditions.

Main result

We know a difference Δ and a set of conditions on the internal state variables Q_i's, such that:

> *If all the conditions are satisfied by the internal state variable in the computation of $H(M)$,*
> *then $H(M) = H(M + \Delta)$.*

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What is a differential path?

Description

- Specifies how the computations of $H(M)$ and $H(M + \Delta)$ are related.
- The differences introduced in the message evolve in the internal state.
- Differential attack with the modular difference.
- Most of the work is modulo 2^{32}, but we also need to control bit differences.

What is a differential path?

Notations

Notations

- Modular difference: $\delta(x, y) = y \boxminus x$
- Wang's difference: $\partial(x, y) = \langle y^{[31]} - x^{[31]}, \ldots y^{[0]} - x^{[0]} \rangle$
- ▲ and ▼ for $+1$ and -1.
- $x^{[k]}$ for the $k+1$-st bit of x.
- Compact notation: $\langle$▲$^{[0]}$, ▼▲$^{[3,4]}$, ▲▲$^{[30,31]}\rangle$

Differential path notations

- We consider a message M. $M' = M \boxplus \Delta$.
- The differential path specifies $\partial Q_i = \partial(Q_i, Q_i')$.
- The desired values are ∂_i.

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Understanding Wang

Question

How to compute the set of conditions?

1. Derive a set of sufficient conditions from a differential path.
2. Compute a differential path from a message difference.
3. Choose a message difference.

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2 Understand and automate
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4 Conclusion

Sufficient conditions computations

Goal

- We are given a differential path $\langle \partial_i \rangle$.
- We want to compute a set of conditions so that:

If $Q(M)$ satisfies the conditions,
then $Q(M)$ and $Q(M')$ follows the path.

Strategy

- We will iteratively add conditions for the current state, assuming the previous ones are satisfied.
- First, study the step operation and the ∂-difference. (Differencial attack)

Remarks about the ∂-difference

The δ-difference and the ∂-difference

- If we know $\partial(x, y)$, we can compute $\delta(x, y)$.
- If we know $\delta(x, y)$, many $\partial(x, y)$ are possible.
  For instance, if $\delta(x, y) = 2^k$, $33 - k$ possibilities:

$$
\begin{array}{ll}
\langle \blacktriangle^{[k]} \rangle & \rightarrow 2^k \\
\langle \blacktriangledown\blacktriangle^{[k,k+1]} \rangle & \rightarrow 2^{k+1} - 2^k \\
 & \dots \\
\langle \blacktriangledown\blacktriangledown\dots\blacktriangledown\blacktriangle^{[k,k+1,\dots 30,31]} \rangle & \rightarrow 2^{31} - 2^{30} - \dots - 2^k \\
\langle \blacktriangledown\blacktriangledown\dots\blacktriangledown\blacktriangledown^{[k,k+1,\dots 30,31]} \rangle & \rightarrow \cancel{2^{32}} - 2^{31} - \dots - 2^k
\end{array}
$$

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Remarks about the ∂-difference

Theorem

$$\partial(x,y) = \langle \varepsilon_{31}, \varepsilon_{30}, \dots \varepsilon_0 \rangle \iff \begin{cases} \sum_{j=0}^{31} \varepsilon_j 2^j = \delta(x,y) \\ \forall j, \varepsilon_j \in \{-1, 0, +1\} \\ \forall j : \varepsilon_j = +1 \implies x^{[j]} = 0 \\ \forall j : \varepsilon_j = -1 \implies x^{[j]} = 1 \end{cases}$$

- If we know $\delta(x,y)$, we can fix one $\partial(x,y)$ by adding some conditions on x.
- We can switch between δ-difference and ∂-difference.

Rotation and modular difference

Four cases

- We have an algebraic expression of the rotation: $u \lll s = \lfloor \frac{u}{2^{32-s}} \rfloor + (2^s u \bmod 2^{32})$
- We can express $\upsilon = \delta(a \lll s, b \lll s)$ from $u = \delta(a, b)$

$$\upsilon = \begin{cases} \upsilon_1 = (u \lll s) & \text{if } a + u < 2^{32} \text{ and} \\ & (a \bmod 2^{32-s}) + (u \bmod 2^{32-s}) < 2^{32-s} \\ \upsilon_2 = (u \lll s) \boxplus 1 & \text{if } a + u < 2^{32} \text{ and} \\ & (a \bmod 2^{32-s}) + (u \bmod 2^{32-s}) \geq 2^{32-s} \\ \upsilon_3 = (u \lll s) \boxminus 2^s & \text{if } a + u \geq 2^{32} \text{ and} \\ & (a \bmod 2^{32-s}) + (u \bmod 2^{32-s}) < 2^{32-s} \\ \upsilon_4 = (u \lll s) \boxminus 2^s \boxplus 1 & \text{if } a + u \geq 2^{32} \text{ and} \\ & (a \bmod 2^{32-s}) + (u \bmod 2^{32-s}) \geq 2^{32-s} \end{cases}$$

$\rightarrow$ bit conditions, probabilities

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Rotation and modular difference

Important remark

- The conditions are on the input (or output) of the rotation.
- In MD4, we will use this backwards:
  $Q_{i+4} = (Q_i \boxplus \Phi_{i+4} \boxplus m_{i+4} \boxplus k_{i+4}) \lll s_{i+4}$

Wang difference and Boolean functions

The Boolean function

- Bitwise Boolean functions:
  - First round:
    $F(x, y, z) = (x \wedge y) \vee (\neg x \wedge z)$
  - Second round:
    $G(x, y, z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$
  - Third round:
    $H(x, y, z) = x \oplus y \oplus z$
- For each bit, if we know the input differences
  we can add conditions to select one output difference.
- Motivation for ∂-difference.

Φ_i conditions

			$F(x,y,z) = \mathrm{IF}(x,y,z)$			$G(x,y,z) = \mathrm{MAJ}(x,y,z)$			$H(x,y,z) = x \oplus y \oplus z$		
∂x	∂y	∂z	$\partial F = 0$	$\partial F = 1$	$\partial F = -1$	$\partial G = 0$	$\partial G = 1$	$\partial G = -1$	$\partial H = 0$	$\partial H = 1$	$\partial H = -1$
0	0	0	✓	✗	✗	✓	✗	✗	✓	✗	✗
0	0	$+1$	$x = 1$	$x = 0$	✗	$x = y$	$x \neq y$	✗	✗	$x = y$	$x \neq y$
0	0	-1	$x = 1$	✗	$x = 0$	$x = y$	✗	$x \neq y$	✗	$x \neq y$	$x = y$
0	$+1$	0	$x = 0$	$x = 1$	✗	$x = z$	$x \neq z$	✗	✗	$x = z$	$x \neq z$
0	-1	0	$x = 0$	✗	$x = 1$	$x = z$	✗	$x \neq z$	✗	$x \neq z$	$x = z$
$+1$	0	0	$y = z$	$y,z = 1,0$	$y,z = 0,1$	$y = z$	$y \neq z$	✗	✗	$y = z$	$y \neq z$
-1	0	0	$y = z$	$y,z = 0,1$	$y,z = 1,0$	$y = z$	✗	$y \neq z$	✗	$y \neq z$	$y = z$
0	$+1$	$+1$	✗	✓	✗	✗	✓	✗	✓	✗	✗
0	-1	$+1$	✗	$x = 0$	$x = 1$	✓	✗	✗	✓	✗	✗
0	$+1$	-1	✗	$x = 1$	$x = 0$	✓	✗	✗	✓	✗	✗
0	-1	-1	✗	✗	✓	✗	✗	✓	✓	✗	✗
$+1$	0	$+1$	$y = 0$	$y = 1$	✗	✗	✓	✗	✓	✗	✗
-1	0	$+1$	$y = 1$	$y = 0$	✗	✓	✗	✗	✓	✗	✗
$+1$	0	-1	$y = 1$	✗	$y = 0$	✓	✗	✗	✓	✗	✗
-1	0	-1	$y = 0$	✗	$y = 1$	✗	✗	✓	✓	✗	✗
$+1$	$+1$	0	$z = 1$	$z = 0$	✗	✗	✓	✗	✓	✗	✗
-1	$+1$	0	$z = 0$	$z = 1$	✗	✓	✗	✗	✓	✗	✗
$+1$	-1	0	$z = 0$	✗	$z = 1$	✓	✗	✗	✓	✗	✗
-1	-1	0	$z = 1$	✗	$z = 0$	✗	✗	✓	✓	✗	✗
$+1$	$+1$	$+1$	✗	✓	✗	✗	✓	✗	✗	✓	✗
-1	$+1$	$+1$	✗	✓	✗	✗	✓	✗	✗	✗	✓
$+1$	-1	$+1$	✓	✗	✗	✗	✓	✗	✗	✗	✓
-1	-1	$+1$	✓	✗	✗	✗	✗	✓	✗	✓	✗
$+1$	$+1$	-1	✓	✗	✗	✗	✓	✗	✗	✗	✓
-1	$+1$	-1	✓	✗	✗	✗	✗	✓	✗	✓	✗
$+1$	-1	-1	✗	✗	✓	✗	✗	✓	✗	✓	✗
-1	-1	-1	✗	✗	✓	✗	✗	✓	✗	✗	✓

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Step operations summary

For each operation, we can add conditions on Q_i to make it behave nicely.
$\rightarrow$ Sufficient conditions algorithm.

Computing sufficient conditions

Goal

At step $i+4$, we have:

$Q_{i+4} = (Q_i \boxplus \Phi_{i+4}(Q_{i+1}, Q_{i+2}, Q_{i+3}) \boxplus m_{i+4} \boxplus k_{i+4}) \lll s_{i+4}$
$Q'_{i+4} = (Q'_i \boxplus \Phi_{i+4}(Q'_{i+1}, Q'_{i+2}, Q'_{i+3}) \boxplus m'_{i+4} \boxplus k_{i+4}) \lll s_{i+4}$

We want $\partial(Q_i, Q'_i) = \partial_i$.

Part one: $\delta(Q_i, Q'_i) = \delta_i$

- Choose $\delta^{\ggg}_{i+4} = \delta(Q_{i+4} \ggg s_{i+4}, Q'_{i+4} \ggg s_{i+4})$ that match $\delta_{i+4} = \delta(Q_{i+4}, Q'_{i+4})$.
  $\rightarrow$ $\lll$-conditions on Q_{i+4}.
- We just need $\Phi'_{i+4} \boxminus \Phi_{i+4} = \delta_i \boxminus \delta^{\ggg}_{i+4} \boxplus \Delta_{i+4}$.
  Choose $\partial(\Phi_{i+4}, \Phi'_{i+4})$.
  $\rightarrow$ Φ-conditions on $Q_{i+1}, Q_{i+2}, Q_{i+3}$

Part two: $\partial(Q_i, Q'_i) = \partial_i$

$\rightarrow$ ∂-conditions on Q_i

Computing sufficient conditions

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At step $i+4$, we have:

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$\rightarrow$ ∂-conditions on Q_i

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At step $i+4$, we have:

$Q_{i+4} = (Q_i \boxplus \Phi_{i+4}(Q_{i+1}, Q_{i+2}, Q_{i+3}) \boxplus m_{i+4} \boxplus k_{i+4}) \lll s_{i+4}$

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SC Algorithm

Result

- SC Algorithm works
- Next step: how to compute the differencial path?

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Absorbing the differences

Important observation

$$
\begin{aligned}
Q_i &= (Q_{i-4} \boxplus \Phi_i(Q_{i-1}, Q_{i-2}, Q_{i-3}) \boxplus m_i \boxplus k_i) \lll s_i \\
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Q_{i+2} &= (Q_{i-2} \boxplus \Phi_{i+2}(Q_{i+1}, Q_i, Q_{i-1}) \boxplus m_{i+2} \boxplus k_{i+2}) \lll s_{i+2} \\
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Q_{i+5} &= (Q_{i+1} \boxplus \Phi_{i+5}(Q_{i+4}, Q_{i+3}, Q_{i+2}) \boxplus m_{i+5} \boxplus k_{i+5}) \lll s_{i+5}
\end{aligned}
$$

- We introduce a difference in Q_i.
- If Φ_i can absorb the difference, it will not multiply.
- It only appears every 4 round, with a rotation.

The trivial path

This is the basis for MD4 differential paths:
absorb the message differences.

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- We introduce a difference in Q_i.
- If Φ_i can absorb the difference, it will not multiply.
- It only appears every 4 round, with a rotation.

The trivial path

This is the basis for MD4 differential paths:
absorb the message differences.

Absorbing the differences

Important observation

$$
\begin{aligned}
Q_i &= (Q_{i-4} \boxplus \Phi_i(Q_{i-1}, Q_{i-2}, Q_{i-3}) \boxplus m_i \boxplus k_i) \lll s_i \\
Q_{i+1} &= (Q_{i-3} \boxplus \Phi_{i+1}(Q_i, Q_{i-1}, Q_{i-2}) \boxplus m_{i+1} \boxplus k_{i+1}) \lll s_{i+1} \\
Q_{i+2} &= (Q_{i-2} \boxplus \Phi_{i+2}(Q_{i+1}, Q_i, Q_{i-1}) \boxplus m_{i+2} \boxplus k_{i+2}) \lll s_{i+2} \\
Q_{i+3} &= (Q_{i-1} \boxplus \Phi_{i+3}(Q_{i+2}, Q_{i+1}, Q_i) \boxplus m_{i+3} \boxplus k_{i+3}) \lll s_{i+3} \\
Q_{i+4} &= (Q_i \boxplus \Phi_{i+4}(Q_{i+3}, Q_{i+2}, Q_{i+1}) \boxplus m_{i+4} \boxplus k_{i+4}) \lll s_{i+4} \\
Q_{i+5} &= (Q_{i+1} \boxplus \Phi_{i+5}(Q_{i+4}, Q_{i+3}, Q_{i+2}) \boxplus m_{i+5} \boxplus k_{i+5}) \lll s_{i+5}
\end{aligned}
$$

- We introduce a difference in Q_i.
- If Φ_i can absorb the difference, it will not multiply.
- It only appears every 4 round, with a rotation.

The trivial path

This is the basis for MD4 differential paths:
absorb the message differences.

Absorbing the differences

Important observation

$$
\begin{aligned}
{\color{red}Q_i} &= (Q_{i-4} \boxplus \Phi_i(Q_{i-1}, Q_{i-2}, Q_{i-3}) \boxplus m_i \boxplus k_i) \lll s_i \\
Q_{i+1} &= (Q_{i-3} \boxplus \Phi_{i+1}({\color{red}Q_i}, Q_{i-1}, Q_{i-2}) \boxplus m_{i+1} \boxplus k_{i+1}) \lll s_{i+1} \\
Q_{i+2} &= (Q_{i-2} \boxplus \Phi_{i+2}(Q_{i+1}, {\color{red}Q_i}, Q_{i-1}) \boxplus m_{i+2} \boxplus k_{i+2}) \lll s_{i+2} \\
Q_{i+3} &= (Q_{i-1} \boxplus \Phi_{i+3}(Q_{i+2}, Q_{i+1}, {\color{red}Q_i}) \boxplus m_{i+3} \boxplus k_{i+3}) \lll s_{i+3} \\
{\color{red}Q_{i+4}} &= ({\color{red}Q_i} \boxplus \Phi_{i+4}(Q_{i+3}, Q_{i+2}, Q_{i+1}) \boxplus m_{i+4} \boxplus k_{i+4}) \lll s_{i+4} \\
Q_{i+5} &= (Q_{i+1} \boxplus \Phi_{i+5}({\color{red}Q_{i+4}}, Q_{i+3}, Q_{i+2}) \boxplus m_{i+5} \boxplus k_{i+5}) \lll s_{i+5}
\end{aligned}
$$

- We introduce a difference in Q_i.
- If Φ_i can absorb the difference, it will not multiply.
- It only appears every 4 round, with a rotation.

The trivial path

This is the basis for MD4 differential paths:
absorb the message differences.

Absorbing the differences

MD4 Boolean functions

$F(x, y, z) = (x \wedge y) \vee (\neg x \wedge z)$

MD4 Boolean function F can absorb one input difference:

$$F(x, y, z) = \mathrm{IF}(x, y, z)$$

∂x	∂y	∂z	$\partial F = 0$	$\partial F = 1$	$\partial F = -1$
0	0	0	✓	✗	✗
0	0	+1	$x = 1$	$x = 0$	✗
0	0	−1	$x = 1$	✗	$x = 0$
0	+1	0	$x = 0$	$x = 1$	✗
0	−1	0	$x = 0$	✗	$x = 1$
+1	0	0	$y = z$	$y, z = 1, 0$	$y, z = 0, 1$
−1	0	0	$y = z$	$y, z = 0, 1$	$y, z = 1, 0$

Absorbing the differences

MD4 Boolean functions

$G(x, y, z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$

MD4 Boolean function G can absorb one input difference:

$$G(x, y, z) = \mathrm{MAJ}(x, y, z)$$

∂x	∂y	∂z	$\partial G = 0$	$\partial G = 1$	$\partial G = -1$
0	0	0	✓	✗	✗
0	0	+1	$x = y$	$x \neq y$	✗
0	0	−1	$x = y$	✗	$x \neq y$
0	+1	0	$x = z$	$x \neq z$	✗
0	−1	0	$x = z$	✗	$x \neq z$
+1	0	0	$y = z$	$y \neq z$	✗
−1	0	0	$y = z$	✗	$y \neq z$

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Absorbing the differences

MD4 Boolean functions

$H(x, y, z) = x \oplus y \oplus z$

MD4 Boolean function H can not absorb one input difference:

$H(x, y, z) = x \oplus y \oplus z$

∂x	∂y	∂z	$\partial H = 0$	$\partial H = 1$	$\partial H = -1$
0	0	0	✓	✗	✗
0	0	+1	✗	$x = y$	$x \neq y$
0	0	−1	✗	$x \neq y$	$x = y$
0	+1	0	✗	$x = z$	$x \neq z$
0	−1	0	✗	$x \neq z$	$x = z$
+1	0	0	✗	$y = z$	$y \neq z$
−1	0	0	✗	$y \neq z$	$y = z$

Note: Wang use a local collision in round 3, no need to search path.

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Differential Path Search

Basic Idea

- Follow the sufficient conditions algorithm.
- $Q_{i+4} = (Q_i \boxplus \Phi_{i+4} \boxplus m_{i+4} \boxplus k_{i+4}) \lll s_{i+4}$
  $Q'_{i+4} = (Q'_i \boxplus \Phi'_{i+4} \boxplus m'_{i+4} \boxplus k_{i+4}) \lll s_{i+4}$
- We do not know ∂Q_i, so we assume $\Phi'_i = \Phi_i$,
  ie. absorb the difference. $\rightarrow \delta^{\ggg}_{i+4} = \delta_i$.
- Goes from the last step to the first.
- When we have a path up to the first round,
  there might be a difference in the IV, we will fix it later.

Differential Path Search

Turning pseudo-collision path into collision path

- We run the algorithm again, using the previous path as a hint for the values of $\delta\Phi_i$.
- We try to modify the path on the bits that will become the IV differences.

Path representation

- During the computation, the path is represented by ∂_i's.
- To modify the path later, we will rather use the $\delta\Phi_i$'s.

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Pseudo-code

```
function PATHFIND
    𝒫 ← {ε}
    loop
        extract P from 𝒫
        PATHSTEP(P,ε,48)
function PATHSTEP(P_0,P,i)
    if i < 0 then
        add P in 𝒫
    else
        for all possible choice P′ do
            PATCHTARGET(P_0,P′,i)
function PATCHTARGET(P_0,P,i)
    for all possible choice P′ do
        PATCHCARRIES(P_0,P′,i)
function PATCHCARRIES(P_0,P,i)
    for all possible choice P′ do
        PATHSTEP(P_0,P′,i − 1)
```

Pseudo-code

```
function PathFind
    𝒫 ← {ε}
    loop
        extract P from 𝒫
        PathStep(P,ε,48)
function PathStep(P_0,P,i)
    if i < 0 then
        add P in 𝒫
    else
        for all possible choice P' do
            PatchTarget(P_0,P',i)
function PatchTarget(P_0,P,i)
    for all possible choice P' do
        PatchCarries(P_0,P',i)
function PatchCarries(P_0,P,i)
    for all possible choice P' do
        PathStep(P_0,P',i − 1)
```

PathFind

- Starts with the trivial path
- Pick a path and try to improve it

Pseudo-code

function PATHFIND
    $\mathcal{P} \leftarrow \{\epsilon\}$
    loop
        extract P from $\mathcal{P}$
        PATHSTEP(P,ϵ,48)

function PATHSTEP(P_0,P,i)
    if $i < 0$ **then**
        add P in $\mathcal{P}$
    else
        for all possible choice P' **do**
            PATCHTARGET(P_0,P',i)

function PATCHTARGET(P_0,
    for all possible choice P' **c**
        PATCHCARRIES(P_0,P'

function PATCHCARRIES(P_0,
    for all possible choice P' **c**
        PATHSTEP(P_0,P',$i-1$

PATHSTEP

- Choose $\delta^{\lll}_{i+4}$ from δ_{i+4} and $\partial\Phi_{i+4}$ from $\delta\Phi_{i+4}$
- Compute δQ_i from $\delta^{\lll}_{i+4}$ and $\partial\Phi_{i+4}$

Pseudo-code

```
function PathFind
    𝒫 ← {ε}
    loop
        extract P from 𝒫
        PathStep(P,ε,48)
function PathStep(P_0,P,i)
    if i < 0 then
        add P in 𝒫
    else
        for all possible choice P' do
            PatchTarget(P_0,P',i)
function PatchTarget(P_0,P,i)
    for all possible choice P' do
        PatchCarries(P_0,P',i)
function PatchCarries(P_0,P,i)
    for all possible choice P' do
        PathStep(P_0,P',i − 1)
```

PatchTarget

- Modify $\partial\Phi_i$ from the path P.

Pseudo-code

```
function PATHFIND
    𝒫 ← {ε}
    loop
        extract P from 𝒫
        PATHSTEP(P,ε,48)
function PATHSTEP(P0,P,i)
    if i < 0 then
        add P in 𝒫
    else
        for all possible choice P′ do
            PATCHTARGET(P0,P′,i)
function PATCHTARGET(P0,P,i)
    for all possible choice P′ do
        PATCHCARRIES(P0,P′,i)
function PATCHCARRIES(P0,P,i)
    for all possible choice P′ do
        PATHSTEP(P0,P′,i − 1)
```

PatchCarries

- Choose ∂Q_i from δQ_i

Correcting the differences

Direct correction

- $Q_i = (Q_{i-4} \boxplus \Phi_i \boxplus m_i \boxplus k_i) \lll s_i$
- Differences do not multiply: each difference in the IV has to be fixed in exactly one place.
- Possible places: every 4 rounds.
- We use Φ_i to modify the bit.

Indirect Corrections

- $Q_{i+a} = (Q_{i+a-4} \boxplus \Phi_{i+a}(Q_i) \boxplus m_i \boxplus k_i) \lll s_i$
- $Q_i = (Q_{i-4} \boxplus \Phi_i \boxplus m_i \boxplus k_i) \lll s_i$
- We use Q_i to modify Q_{i+a-4}.
- This indroduces a new difference in Q_{i-4}.
- Hopefully, the new difference is easier to remove...

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Message difference

Message difference

- We can try many message differences and run the algorithm
- Interesting message differences depend on the application...

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Overview of the algorithm

Advantages of indirect corrections

- No need to manually add some differences.
- Use freedom in Φ rather than carry expensions.
- Fewer conditions.

Adaptation to MD5?

- $Q_i = Q_{i-1} \boxplus (Q_{i-4} \boxplus \Phi_i(Q_{i-1}, Q_{i-2}, Q_{i-3}) \boxplus m_i \boxplus k_i) \lll s_i$
- No easy way to stop difference multiplications.
  Use den Boer-Bosselaers's path?
- No easy way to express the rotation conditions.

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Collisions

Collision path

- We want to minimize the search complexity
- Few conditions in 3rd (and 2nd) round: local collision.
- Our algorithm works with Wang's message difference, not (yet?) with Sasaki *et al.*'s.

Comparison of collision paths

Number of conditions	round 1	round 2	round 3	total
With Wang's message difference:				
Wang *et al.*	96	25	2	123
Schläffer and Oswald	122	22	2	146
Our path	72	16	2	90
With Sasaki's message difference:				
Sasaki *et al.*	167	9	1	177

Second preimage

Second preimage paths

- Second preimage for weak message
- If c conditions, a message is weak with probabilty 2^{-c}
- We want to minimize the number of conditions

Results on Yu's path

- Yu *et al.* gave a path with one bit difference in m_4
- Authors claim 32 path using rotations of the path. Actually, only 28 paths (fails on bit 17,20,26 and 28).
- Using bit 25, only 58 conditions instead of 62. Good if you need only one path with very few conditions (eg. Contini Yin HMAC-MD4 attacks).

A New NMAC Attack

Main idea

- We search for a differential path with the message difference in m_0:

step	s_i	δm_i	$\partial\Phi_i$	∂Q_i	conditions
0	3	$\langle \blacktriangle^{[0]} \rangle$		$\langle \blacktriangle^{[3]} \rangle$	
1	7				$Q_{-1}^{[3]} = Q_{-2}^{[3]}$ (X)
2	11				$Q_1^{[3]} = 0$
3	19				$Q_2^{[3]} = 1$
4	3			$\langle \blacktriangle^{[6]} \rangle$	

- The beginning of the path depends on a condition (X) of the IV.
- $\Pr[H(M) = H(M+\Delta) | X] = p \gg 2^{-128}$.
- $\Pr[H(M) = H(M+\Delta) | \neg X] \approx 2^{-128}$.
- We learn one bit of the IV with about $2/p$ message pairs.

A New NMAC Attack

How to recover the outer key

NMAC Description

- $\mathrm{NMAC}_{k_1,k_2}(M) = H_{k_1}(H_{k_2}(M))$
- To recover k_1, we have to control $H_{k_2}(M)$.
- We need about $2/p$ message pairs such that $H_{k_2}(M_2) = H_{k_2}(M_1) + \Delta$.
- Δ must be only in the first 128 bits.
- We can use the birthday paradox: we need to hash about $2^{\frac{n - \log p}{2}}$ messages.

Advantage

- In Contini-Yin attack, you need to control the value of $H_{k_2}(M)$ (related messages).
- We only need to control the differences of $H_{k_2}(M)$.

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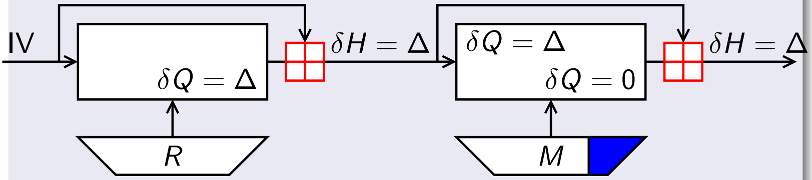
A New NMAC Attack

How to recover the outer key

Efficient computation of message pairs

- We start with *one* message pair (R_1, R_2) such that $H_{k_2}(R_2) = H_{k_2}(R_1) + \Delta$ (birthday paradox).
- We compute second blocks (M_1, M_2) such that $H_{k_2}(R_2||M_2) = H_{k_2}(R_1||M_1) + \Delta$
- This is essentially a collision search with the padding inside the block.



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The Attack against NMAC-MD4

Differential paths

- We need paths with a difference in m_0 and no difference in $m_4 \ldots m_{15}$.
- We found 22 paths with one bit difference in m_0 and $p \approx 2^{-79}$.
- Unlikely to find such paths in MD5.

Complexity

- We can recover the full NMAC key (k_1, k_2)
- 2^{88} online request to the NMAC oracle.
- 2^{105} offline hash computations.
  2^{94} by using more than one bit of information per path.

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Future work

Improving the algorithm

- Using ideas from Stevens *et al.* and Sasaki *et al.*...

Other uses

- Try to find new kind of attack based on new types of path...