- Large integers should be factored more rapidly. Maple 9 can factor some faster.
- Expressions involving Jacobi elliptic functions are not simplified using the standard identities.
- A single equals sign should not act as an assigning operator inside `DSolve` commands.

Despite these relatively minor problems, version 5.0 is a significant improvement over previous versions. *Mathematica 5.0*'s support of sparse matrices, differential-algebraic equations, difference equations, inequalities, and improved speed make the upgrade worthy of serious consideration.

The suggested retail price for *Mathematica 5.0*, including one year of Premier Service, is $1880. The academic and student versions list for $895 and $139.95, respectively. Both these versions have the same functionality as the professional version. *Mathematica 5.0* can be purchased directly at www.wolfram.com.

REFERENCES

[1] M. L. Abell and J. P. Braselton, *Mathematica by Example*, 3rd ed., Academic Press, New York, 2003.

[2] J. W. Gray, *Mastering Mathematica: Programming Methods and Applications*, Academic Press, New York, 2003.

[3] E. Packell, *Mathematica for Mathematics Teachers: Notes from an Introductory Course*, Front Range Press, 1996.

John D. Carter
Seattle University

Numerical Optimization. *By J. Frédéric Bonnans, J. Charles Gilbert, Claude Lemaréchal, and Claudia A. Sagastizábal.* Springer-Verlag, Berlin, 2003. $54.95. xiv+419 pp, softcover. ISBN 3-540-00191-3.

This book is a translation and slight revision of the French version [1] written by the same set of authors and published in 1997. That work, in turn, is based on a set of lecture notes prepared for an intensive course offered several times in France and other countries. In addition, the authors have taught this material at various *Grandes Ecoles*. According to the authors' declaration, "the expected readers are engineers, Master or Ph.D. students, confirmed researchers, in applied mathematics or from various other disciplines where optimization is a need." All this is to suggest that the book has been classroom tested.

Yet this is not a textbook for students with no background in optimization whatsoever. Indeed, the authors plainly state that "little is said concerning optimization theory proper (optimality conditions, constraint qualification, stability theory)." Knowledge of these matters and an awareness of the various classes of optimization problems and their applications is essential to an appreciation of what is conveyed here, namely, "numerical algorithms for optimization, their theoretical foundations and convergence properties, as well as their implementation, their use, and other practical aspects." As the authors put it, "we keep as a leading thread the *practical value* of optimization methods, in terms of their efficiency to solve real-world problems." In this respect, one thinks of the monographs by Fletcher [2], Gill, Murray, and Wright [4], and Nocedal and Wright [6]. As compared with these three works, the book of Bonnans et al. is, I believe, the most demanding from the mathematical standpoint. It is probably also the most emphatic with regard to what is useful. For instance, in speaking of a standard version of the steepest-descent method, Lemaréchal writes, "this method is *very bad* because it is very slow; ... [it] should actually be **forbidden**."

This English language version of the book is approximately 100 pages longer than its French parent. One has to estimate the difference in length because the French edition has slightly greater line length (`\textwidth`.) The greater length is the result of splitting one chapter into two, the inclusion of new sections in the aforementioned chapters as well as others, and the introduction of comments and exercises. In reading this book, one has to make allowances for the spotty English usage. Most of the time, the text is perfectly clear and correct. But every once in a while the fact that it was not written by anglophones is obvious. Sometimes the errors are rather amusing, especially when nonexistent words are used, as in the following example: “Extrapolating the proof of Theorem 5.6, it can be thought (a dary [sic] thought, indeed). . . .”

The structure of this work is a bit unusual. It consists of 24 chapters. One of these is devoted to preliminaries (and is so titled); the rest are distributed among four parts, the themes of which are, in order, unconstrained problems (5 chapters), nonsmooth optimization (4 chapters), Newton’s methods in constrained optimization (6 chapters), and interior-point algorithms for linear and quadratic optimization (8 chapters). Each of these four parts is identified with a different one of the four authors. (That is to say, there is a bijection from the set of parts to the set of authors.) Although the parts can be construed as four distinct books lying between just two covers, they are in fact more closely related than that; the authors took a global view of what they were doing and included many references across the porous boundaries between the four parts of their book. They furnish a reference list of 319 entries and an index, primarily concerned with subjects rather than names.

The authorship of the introduction (Chapter 1) is left unstated, although I suspect it is mainly the work of Claude Lemaréchal, to whom the other three authors give thanks for “having given the impetus to this new work by providing a first English version.” Much of the content of this chapter (notations, general problem statements, classification of problems, etc.) is standard, but it features a distinctive selection of motivating examples coming from (a) molecular biology, (b) meteorology, (c) control of a deepwater vehicle, and (d) electric power management. These engaging real-world problems are then keyed to portions of the book where the appropriate “principles of resolution” are discussed. The introduction contains a brief and ultimately dismissive section on Zangwill’s convergence theory. The section states a convergence result from [8] about which the authors say, “checking its hypotheses is in general just as difficult as finding a direct proof [of convergence], ‘case by case’. This section can therefore be skipped.” And in the bibliographical comments at the end of the chapter, they assert that “convergence theory *à la* Zangwill . . . is no longer much used in practice.” I cannot quarrel with this assessment, but the deliberate inclusion of the section and these remarks seem unnecessary.

Part I (by Lemaréchal) is devoted to unconstrained optimization of smooth functions, by which is meant twice continuously differentiable functions or, at the very least, those with Lipschitz continuous gradients. The usual topics are considered: a brief—and disparaging—treatment of the gradient method (steepest descent), line search techniques (“of crucial importance in practice”), second-order methods such as Newton’s method (damped and otherwise), and the “utmostly [sic] important and universally used quasi-Newton method” including the well-known BFGS method. Part I also contains a chapter on the conjugate gradient method (for historical reasons, because it “has been much used but is now out of date”) and a final chapter on special methods among which are trust-region techniques, the Gauss–Newton approach to least-squares problems, etc. Trust-region methods and variants of Newton’s principle are identified as methods that “will become classical in the future.”

Part II (by Sagastizábal) is on nonsmooth optimization (NSO), which, as construed here, deals with optimization problems in which the objective function is not continuously differentiable. (The same may also be true of the constraints as well.) Instead, the objective function is here assumed to be convex and finite everywhere. Such a function is necessarily continuous and locally Lipschitzian. In this class of problems, the concept of subdifferential plays a central role.

It is in this part of the book that the greatest difference between the English and French editions is to be found. Its opening chapter is concerned with theoretical aspects of nonsmooth optimization. After a section on convex analysis, there come two new sections. The first is on Lagrangian relaxation and duality, and the second is on two special nondifferentiable convex functions (the pointwise maximum of a collection of smooth convex functions and the Lagrangian dual function). Next comes a chapter on special methods in NSO. A few enlightening examples illustrate how the naive transfer of techniques from smooth optimization to nonsmooth optimization may lead to serious computational traps. Descent methods and "black box" methods round out the chapter. Part II concludes with a chapter on bundle methods followed by another on decomposition and duality.

In Part III we come to constrained optimization via Newton-like methods. To a large extent, this amounts to using the methodology known as sequential quadratic programming (SQP). In the generally useful introduction to this part, the author (Gilbert) states that "the basic idea is to linearize the optimality conditions of the problem and to express the resulting linear system in a form suitable for calculation." At first glance, this view of the procedure seems at variance with the usual interpretation, which speaks of (the successive) quadratic approximation of the objective function and linear approximation of the constraints after which the optimality conditions of the resulting quadratic program are to be solved. But the two approaches amount to the same thing.

Recognizing that the subject at hand can raise many questions and take many directions for their answers, Gilbert wisely observes that "there is little to be gained from describing each of these algorithms." His aim, instead is "to present the concepts that form the building blocks of these methods and to show why they are relevant." In a small way, this remark typifies the very nice pedagogical orientation of this part of the book. Another fact that distinguishes this part of the book from the rest is that five of its six chapters end with sets of exercises. The content of Part III includes a worthwhile chapter on background material with sections on first- and second-order optimality conditions, speed of convergence, projection onto a closed convex set, and Newton's method. Other chapters deal with equality-constrained problems, problems with mixed constraints (equations and inequalities), exact penalization techniques, and quasi-Newton methods.

Part IV (by Bonnans) covers interior-point algorithms for linear and quadratic programming. It delivers an introduction to the simplex algorithm and a treatment of primal-dual path-following methods. The subject is developed in the framework of monotone linear complementarity problems, as in the work of Kojima et al. [5] and S. J. Wright [7], among others. Those familiar with this subject will recognize the inadequacy of packing such a large body of knowledge into such a tiny nutshell.

The opening chapter of this part begins with an existence theorem for linearly constrained *convex* optimization problems. The constraints are expressed in standard form: linear equations in nonnegative variables. A part of this theorem can be viewed as a restrictive form of the Frank–Wolfe theorem for quadratic programming (see [3, Appendix I], which is not cited in this book). This is followed by a section on (Lagrangian) duality for linear and quadratic programming, the concept of saddle-point, and the Goldman–Tucker theorem on strict complementarity in linear programming. Finally, we come to the simplex algorithm—hardly an interior-point algorithm, but one that is of historical importance, provides a benchmark for algorithmic analysis, and is of value in the "purification process" by which an "exact solution" is computed. As may be expected, the development of the simplex algorithm is matrix theoretic.

In a chapter on "linear monotone complementarity" (or monotone linear complementarity) and associated vector fields, Bonnans develops theoretical tools for the subsequently presented algorithms. Among these tools are the logarithmic penalty function, the concept of the central path, properties of the monotone LCP (including some interesting and seldom discussed group-theoretic results), vector fields associated with the central path, and continuous trajectories where the differential equations arising in the aforementioned vector fields are considered.

The next two chapters briefly cover predictor-corrector algorithms and non-feasible algorithms, respectively, along with a bit of the corresponding complexity analysis.

Self-duality is the subject of the following chapter. The emphasis is on the *form* of the LCP that arises in the study of linear programs with linear inequality constraints in nonnegative variables. Much is made of this property in path-following methods. It must be said that the subject of self-duality goes back much further than one would ever imagine from reading this chapter.

The last three chapters of Part IV cover one-step methods, the complexity of linear optimization with integer data, and finally a very short (and antiseptic) discussion of Karmarkar's algorithm. In the latter chapter (as in others) much of the scholarship one would expect to find in a work such as this is conveyed only through the reference lists of a few cited journal articles.

When viewed as the lecture notes of an intensive course, this book is remarkably fine. It covers a lot of ground in a lively, authoritative way. But in reading it, I repeatedly asked myself the question, Could I use this as a textbook in one of my graduate courses on optimization? I consistently responded, "No, not all by itself." It is too abstract, too lacking in numerical examples and exercises. Parts of it would make excellent collateral reading. Parts of it would serve nicely as the skeleton to which some flesh could be attached to create a course for graduate students in engineering and the mathematical sciences, especially those in North America whose undergraduate background provides so little exposure to mathematical rigor and abstraction. Of course, as noted at the outset, the audience envisioned by the authors is broader than just students. This book definitely merits a place on the bookshelf of the "confirmed researcher."

REFERENCES

[1] J. F. Bonnans, J. C. Gilbert, C. Lemaréchal, and C. Sagastizábal, *Optimisation Numérique*, Springer-Verlag, Berlin, 1997.

[2] R. Fletcher, *Practical Methods of Optimizations*, Vols. 1 and 2, John Wiley, Chichester, UK, 1980.

[3] M. Frank and P. Wolfe, *An algorithm for quadratic programming*, Naval Res. Logistics Quart., 3 (1956), pp. 95–110.

[4] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization*, John Wiley, London, 1981.

[5] M. Kojima, N. Megiddo, T. Noma, and A. Yoshise, *A Unified Approach to Interior-Point Algorithms for Linear Complementarity Problems*, Lecture Notes in Comput. Sci. 538, Springer-Verlag, Berlin, 1991.

[6] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer-Verlag, New York, 1999.

[7] S. J. Wright, *Primal-Dual Interior-Point Methods*, SIAM, Philadelphia, 1997.

[8] W. I. Zangwill, *Nonlinear Programming*, Prentice-Hall, Englewood Cliffs, NJ, 1969.

Richard W. Cottle
Stanford University

Computational Mathematics: Models, Methods, and Analysis with Matlab and MPI. *By R. E. White*. Chapman & Hall/CRC, Boca Raton, FL, 2004. $89.95. xvi+385 pp., hardcover. ISBN 1-58488-364-2.

To quote from the preface:

> This book evolved from the need to migrate computational science into undergraduate education. It is intended for students who have had basic physics, programming, matrices and multivariable calculus.
>
> The choice of topics in the book has been influenced by the Undergraduate Computational Engineering and Science project (a United States Department of Energy funded effort). . . .

This textbook shows how to numerically solve relatively simple partial differential equations arising from the modeling of physical situations such as heat transfer and fluid flow. It uses Matlab and Fortran 90 (with MPI for parallel programming). The Matlab and Fortran 90 codes emphasize vectorization of the linear algebra; indeed, it is the only feature specific to Fortran 90 that is exploited widely.

The book falls into two parts. In the first, the basic ideas of physical modeling are used to derive the partial differential equations describing physical situations and their discretizations. This material is interspersed with developments of standard numerical methods mainly associated with the