New perspectives in Differential Cryptanalysis of SPN's

Merlin Fruchon [1,2], Anne Canteaut[1]

[1]Inria, France, [2]DGA, France

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Outline

Plan of this Section

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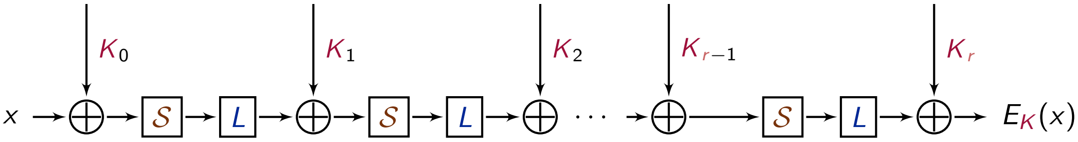
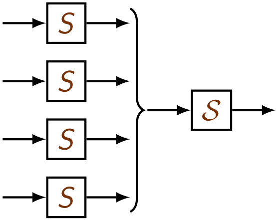
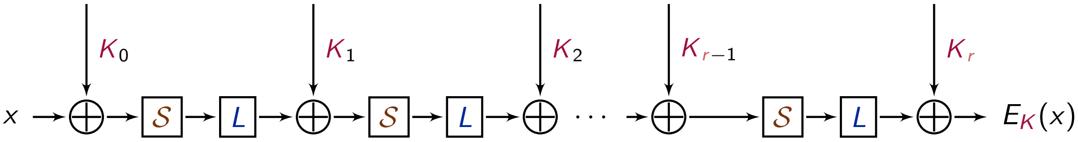
Substitution Permutation Networks



Substitution Permutation Networks





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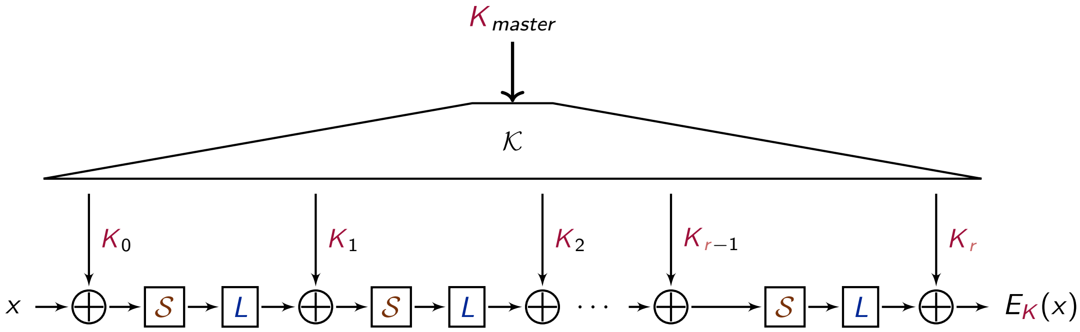
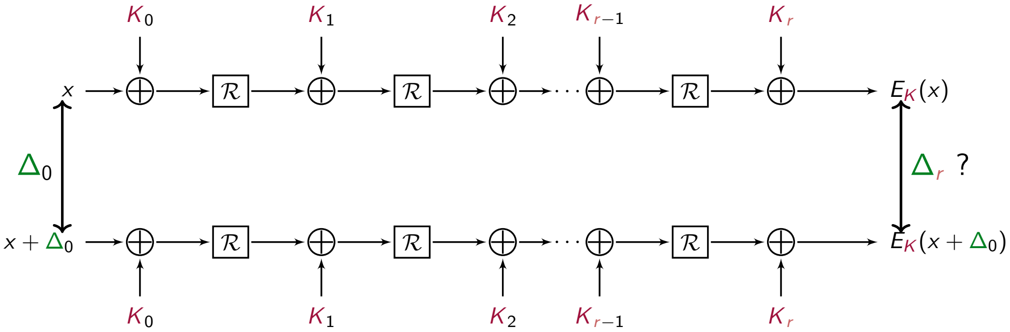


Plan of this Section

Differentials [BS91]

$\mathcal{R} = L \circ \mathcal{S}$



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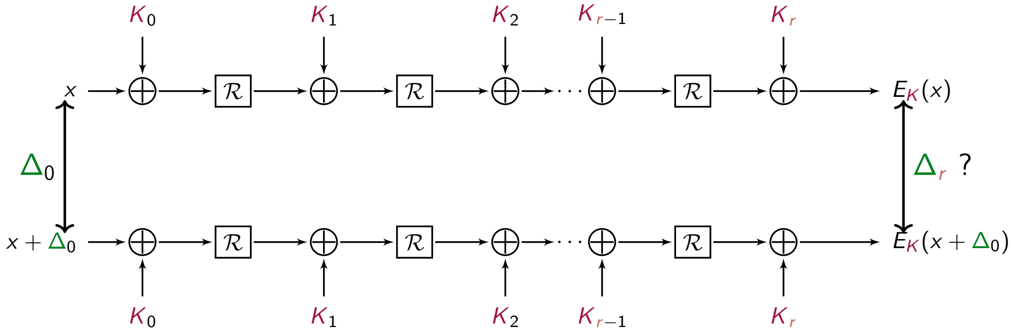
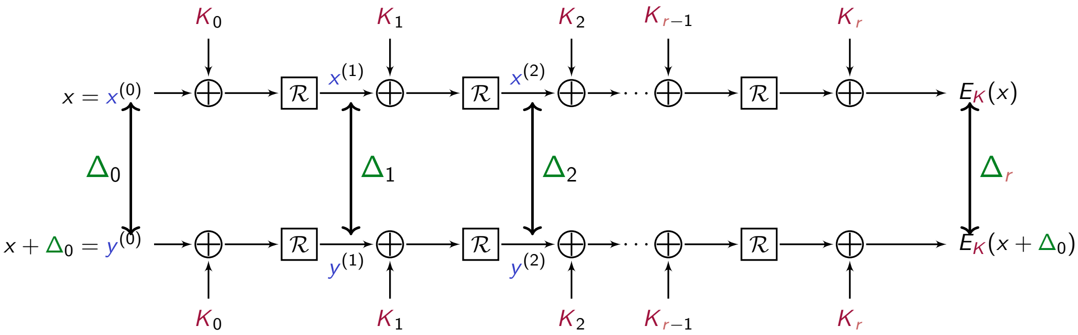


Probability of a differential

$$\mathbb{P}(K, \Delta_0, \Delta_r) := \frac{\#\{x \in \mathbb{F}_2^n \mid E_K(x + \Delta_0) + E_K(x) = \Delta_r\}}{2^n}$$

Differential characteristics



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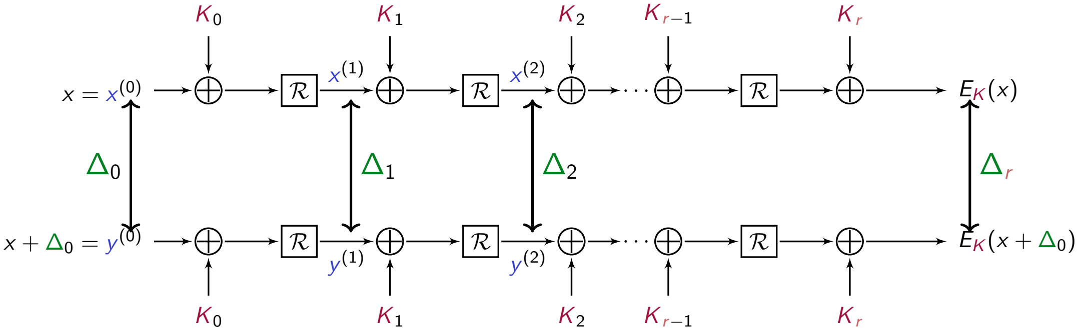

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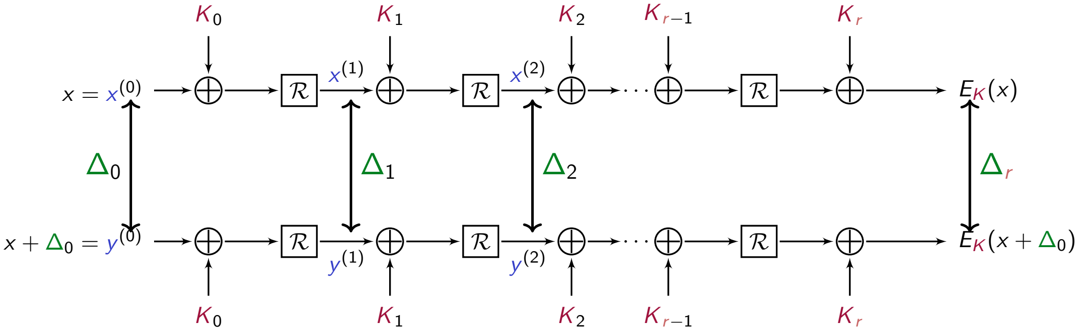

Probability of a differential characteristic

$$\mathbb{P}(K, \Delta_{0\leq j\leq r}) := \frac{\#\{x \in \mathbb{F}_2^n \mid \forall j \in [\![0, r]\!], x^{(j)} + y^{(j)} = \Delta_j\}}{2^n}$$

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Usual assumptions [LMM91]

Expected Differential Probability

$$EDP(\Delta_0, \Delta_r) := \frac{1}{2^n} \sum_{K \in \mathbb{F}_2^n} \mathbb{P}(K, \Delta_0, \Delta_r)$$

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1. Stochastic equivalence : $\forall K, \mathbb{P}(K, \Delta_0, \Delta_r) \approx EDP(\Delta_0, \Delta_r)$
2. Round keys are independent and uniformly distributed
3. Dominant trail: the probability of the differential is close to the probability of the most probable characteristic.

Validity of these assumptions

- For the DES, $EDP = 2^{-47.22}$ but equal to $2^{-43.16}$ for some keys and $2^{-55.16}$ for other keys [Knu93].

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Validity for lightweight designs

In $\textsc{Midori64}$, for some weak keys there are characteristics of probability $> 2^{-32}$ for any number of rounds.

Plan of this Section

A framework to study fixed-key behavior [BR22]

Quasi-differential framework

- Analyzes the fixed-key behavior of differential characteristics.

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Drawback

Matrices of size $2^{2^n} \times 2^{2^n}$ hence computation infeasible in general.

Plan of this Section

Generalization of planar pairs : [DR07]

Super planar pairs

(a, b) is a super planar pair for an Sbox S if

$$\left\{ \begin{pmatrix} x \\ S(x) \end{pmatrix} \mid S(x) + S(x+a) = b \right\}$$

is an affine space.

Remark

If $\# \{x \mid S(x) + S(x+a) = b\} \in \{2, 4\}$, (a, b) is super planar.

Our starting point

Consider a SPN consisting of r rounds with linear layer L and a non-linear layer which applies S in parallel.

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Theorem

Let Δ be a differential characteristic such that, for each intermediate Sbox, the pair (input difference, output difference) is super planar. Then :

$$\mathbb{P}(K, \Delta) = \begin{cases} p_0 \times 2^{\dim(W)} & \text{if } K \in c + W^{\perp} \\ 0 & \text{otherwise} \end{cases}$$

- p_0 is the probability that we obtain using the usual assumptions.
- $W \subset \mathbb{F}_2^{n(r-1)}$ is a vector space depending on Δ, L and S.
- $c + W^{\perp}$ is an affine subspace whose linear part is $W^{\perp}$.

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Confirming known (but surprising) results

- MIDORI64 is a light-weight block cipher proposed by [BBI+15]
- Linear key-schedule

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- Linear key-schedule

Baudrin et al. proved the following result using *commutative cryptanalysis*.

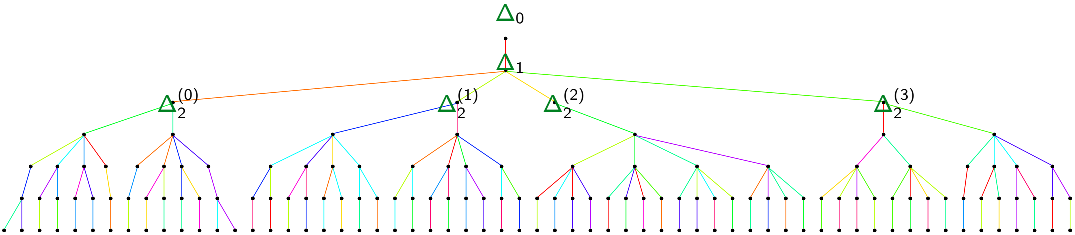
Proposition [BFL+23]

Let $\Delta_0 \in \{\texttt{0xa},\texttt{0xf}\}^{16}$ and $K_i \in \langle \texttt{0x2},\texttt{0x5},\texttt{0x8}\rangle^{16}$. Then for any r,

$$\sum_{\Delta_r \in \{\texttt{0xa},\texttt{0xf}\}^{16}} \mathbb{P}(K, \Delta_0, \Delta_r) \geq 2^{-16}$$

Our framework allows us to confirm and extend this result.

How we thought it behaved



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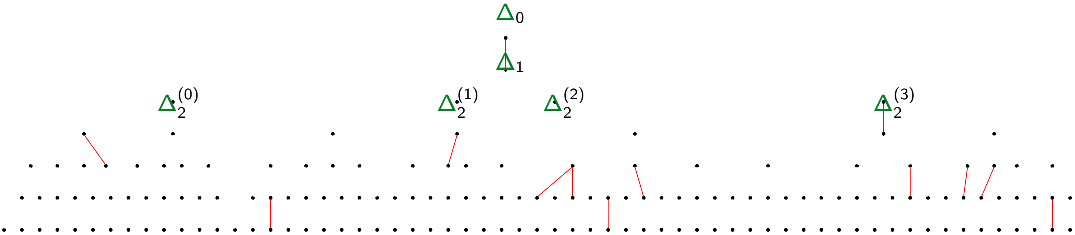
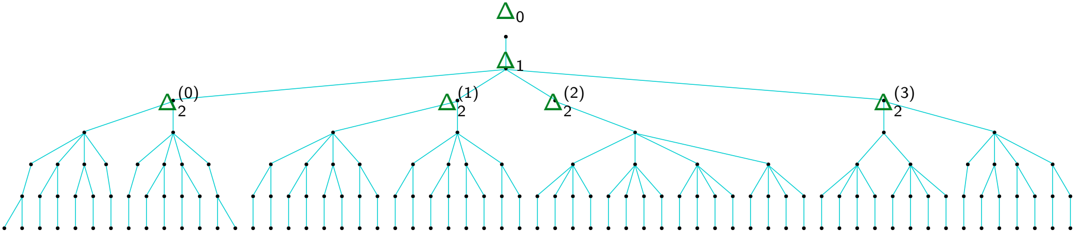


How it really behaves



All characteristics are valid for the same keys !

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- The result holds for modified MIDORI64 with non-involutive Sboxes.
- Our framework confirms a similar result on SCREAM.

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- Our work gives a partial answer.

Some more applications

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Thank you for listening !

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