

Cryptanalysis of Full Sprout

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Virginie Lallemand and María Naya-Plasencia

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New Design Principle



Frederik Armknecht, Vasily Mikhalev

On Lightweight Stream Ciphers with Shorter Internal States,
FSE 2015

Background: Lightweight Cryptography

Important metrics:

- area size
- throughput
- memory
- power consumption

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- throughput
- memory
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Block or Stream ciphers?

Throughput generally better for stream ciphers, area size generally worse

Background: Sprout Motivation

Common Design Rule of Stream Ciphers

To avoid **time-memory-data trade-off attacks**, the internal state size of a stream cipher should be at least twice the security parameter

→ Larger area size, bigger power consumption

Objective of the new stream cipher design:

Reduce the size of the internal state used in stream ciphers while offering resistance to time-memory-data trade-off attacks

New Design Principle

- Sufficiently large internal state
- Part of this state is the secret key itself
- Involve the secret key not only in the initialization process but also in the keystream generation phase

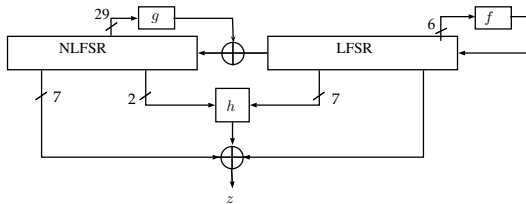
Remark

Storing a fixed key is significantly less area consuming than deploying a register of the same length

Concrete instantiation of this new design strategy: **Sprout**

Description of Sprout

Grain Family



Keystream Generation Phase of Grain-128

Grain v1

80-bit LFSR + 80-bit NLFSR

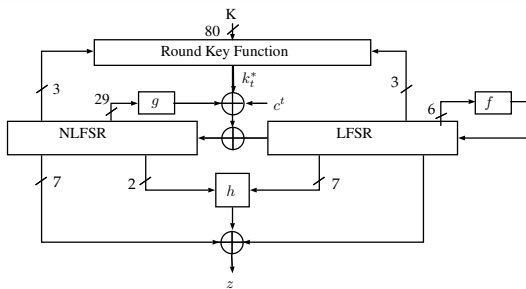
80-bit key

Grain 128 / Grain 128a

128-bit LFSR + 128-bit NLFSR

128-bit key

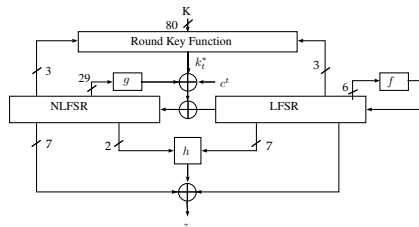
Sprout



Keystream Generation Phase of Sprout

40-bit LFSR + 40-bit NLFSR
80-bit key

Sprout - Keystream Generation Phase



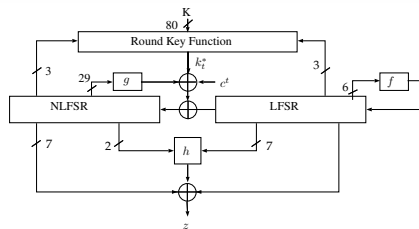
Feedback function of the **LFSR** (maximal period)

$$l_{39}^{t+1} = f(L^t) = l_0^t + l_5^t + l_{15}^t + l_{20}^t + l_{25}^t + l_{34}^t$$

Feedback function of the **NLFSR**

$$\begin{aligned} n_{39}^{t+1} &= n_0^t + n_{13}^t + n_{19}^t + n_{35}^t + n_{39}^t + n_2^t n_{25}^t + n_3^t n_5^t + n_7^t n_8^t + n_{14}^t n_{21}^t \\ &\quad + n_{16}^t n_{18}^t + n_{22}^t n_{24}^t + n_{26}^t n_{32}^t + n_{33}^t n_{36}^t n_{37}^t n_{38}^t + n_{10}^t n_{11}^t n_{12}^t + n_{27}^t n_{30}^t n_{31}^t + c^t + l_0^t + k_t^* \\ &= g(N^t) + c^t + l_0^t + k_t^* \end{aligned}$$

Sprout - Keystream Generation Phase



Feedback function of the **NLFSR**

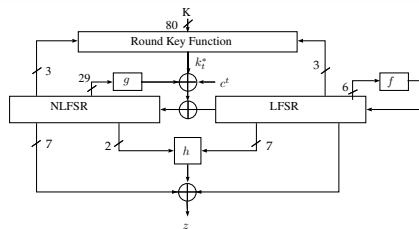
$$n_{39}^{t+1} = g(N^t) + c^t + l_0^t + k_t^*$$

with:

$$k_t^* = k_t, 0 \leq t \leq 79$$

$$k_t^* = (k_{t \bmod 80}) \times (l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t), t \geq 80$$

Sprout - Keystream Generation Phase



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Generation of the keystream bit:

$$z_t = n_4^t l_6^t + l_8^t l_{10}^t + l_{32}^t l_{17}^t + l_{19}^t l_{23}^t + n_4^t n_{38}^t l_{32}^t + l_{30}^t + n_1^t + n_6^t + n_{15}^t + n_{17}^t + n_{23}^t + n_{28}^t + n_{34}^t$$

Our Attack

Overview

Model:

Known Plaintext Scenario: plaintext and ciphertext are known, so we know the value of some keystream bits

We exploit:

- the **short size** of the LFSR and NLFSR register w.r.t the key
- the **little dependencies** between the LFSR and NLFSR registers
- the **non linear influence of the key bits** in the update function

Overview

3 main steps:

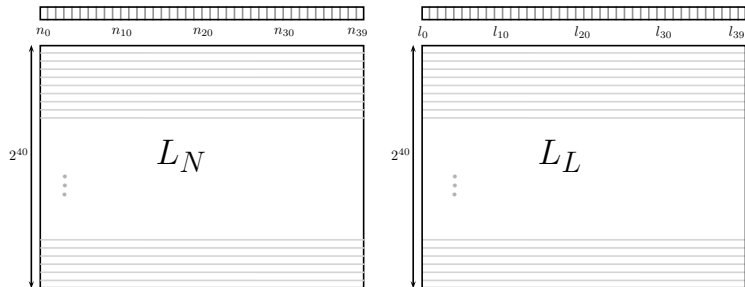
- 1 Build and arrange two independent lists :
 - the possible internal states for the LFSR at time t
 - the possible internal states for the NLFSR at time t
- 2 Merge them with the help of some keystream bits
- 3 Recover the whole key

→ 2^{10} times faster than exhaustive search

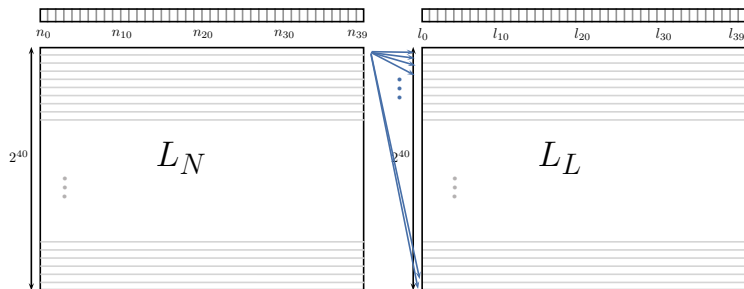
Overview

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Step 1 and 2



Step 1 and 2



Question: How to perform an efficient merge that provides a limited set of candidates for both states?

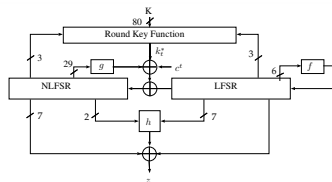
How to Merge

Remark

Property of the linear register

The linear register state is **totally independent** from the rest during the keystream generation phase so its value can be determined at any time

$$l_{39}^{t+1} = l_0^t + l_5^t + l_{15}^t + l_{20}^t + l_{25}^t + l_{34}^t$$



Merging Conditions - Type I and Type II filters

Given an internal state candidate for the NLFSR and for the LFSR at time t

Main idea: Compute z from the candidate and compare it with the actual keystream bit

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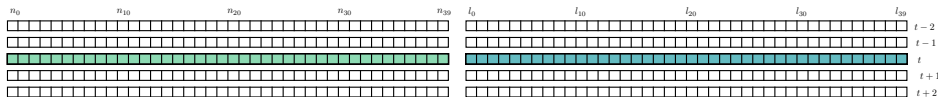
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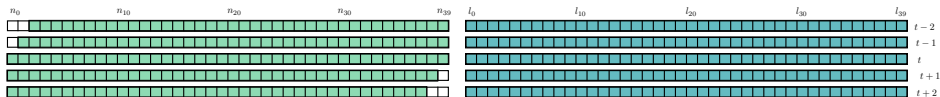


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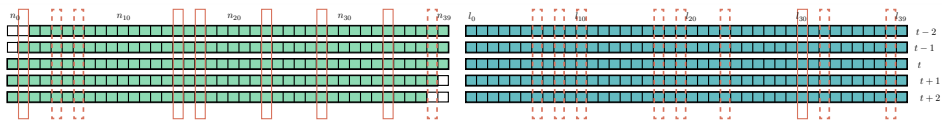


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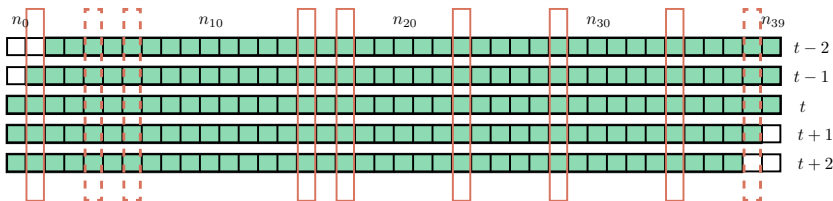


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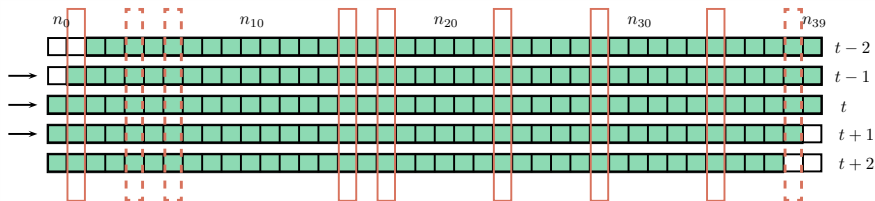


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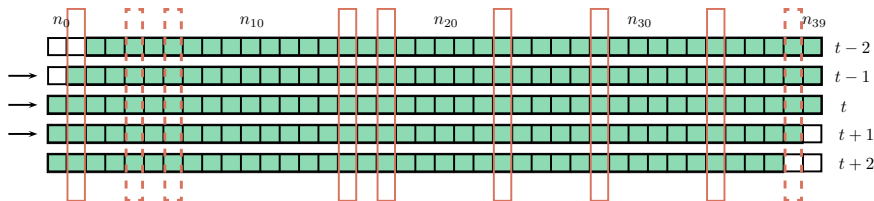


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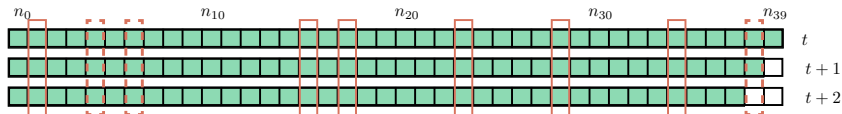
→ check equality at time t , $t + 1$ and $t - 1$
 → **probability of 2^{-3} that a candidate verifies this**

Merging Conditions - Type III filter

Given an internal state candidate for the NLFSR and for the LFSR at time t

How can we use more keystream bits to filter out?

$$z_{t+2} = n_4^{t+2} l_6^{t+2} + l_8^{t+2} l_{10}^{t+2} + l_{32}^{t+2} l_{17}^{t+2} + l_{19}^{t+2} l_{23}^{t+2} + n_4^{t+2} n_{38}^{t+2} l_{32}^{t+2} + l_{30}^{t+2} + n_1^{t+2} + n_6^{t+2} \\ + n_{15}^{t+2} + n_{17}^{t+2} + n_{23}^{t+2} + n_{28}^{t+2} + n_{34}^{t+2}$$

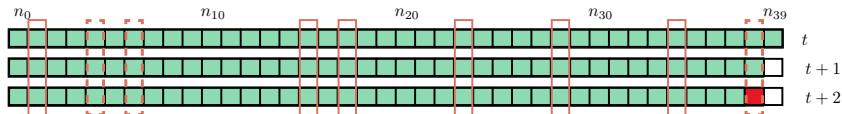


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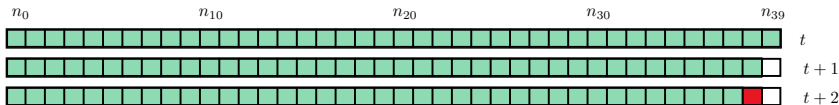
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$$n_{38}^{t+2} = n_{39}^{t+1} = g(N^t) + k_t^* + l_0^t + c^t$$



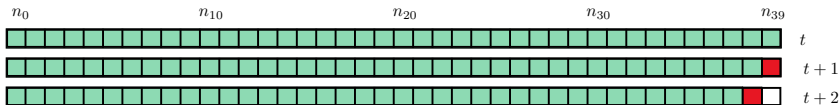
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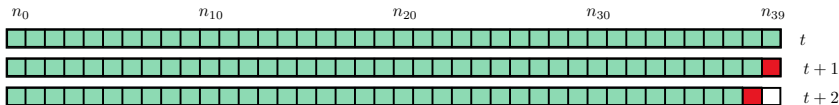
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$$n_{38}^{t+2} = n_{39}^{t+1} = g(N^t) + k_t^* + l_0^t + c^t$$

$$n_{39}^{t+1} = g(N^t) + (k_t \bmod 80) \times (l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t) + l_0^t + c^t$$



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$$n_{39}^{t+1} = g(N^t) + (k_{t \bmod 80}) \times (l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t) + l_0^t + c^t$$

$l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t = 0$	$l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t = 1$
no need of $k_{t \bmod 80}$ compute n_{39}^{t+1} deduce z_{t+2} compare with the actual one	guess $k_{t \bmod 80}$ compute n_{39}^{t+1} deduce z_{t+2} compare with the actual one
probability of being kept: 1/2	probability of being kept: $2 \times 1/2 = \mathbf{1}$

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$l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t = 0$ no need of $k_{t \bmod 80}$ compute n_{39}^{t+1} deduce z_{t+2} compare with the actual one probability of being kept: 1/2	$l_4^t + l_{21}^t + l_{37}^t + n_9^t + n_{20}^t + n_{29}^t = 1$ guess $k_{t \bmod 80}$ compute n_{39}^{t+1} deduce z_{t+2} compare with the actual one probability of being kept: $2 \times 1/2 = \mathbf{1}$
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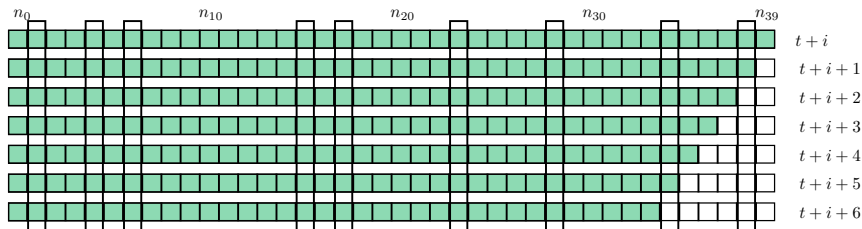
Total probability that a candidate state is kept: $1/2 \times \mathbf{1/2} + 1/2 \times \mathbf{1} = 3/4 = 2^{-0.415}$

Merging Conditions - Type IV filter

Given an internal state candidate for the NLFSR and for the LFSR at time t

$$z_t = n_4^t l_6^t + l_8^t l_{10}^t + l_{32}^t l_{17}^t + l_{19}^t l_{23}^t + n_4^t n_{38}^t l_{32}^t + l_{30}^t + n_1^t + n_6^t + n_{15}^t + n_{17}^t + n_{23}^t + n_{28}^t + n_{34}^t$$

$$n_4^t l_{32}^t = 0 \text{ with probability } 3/4$$

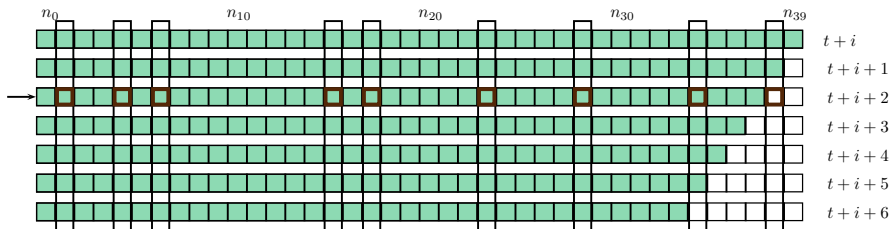


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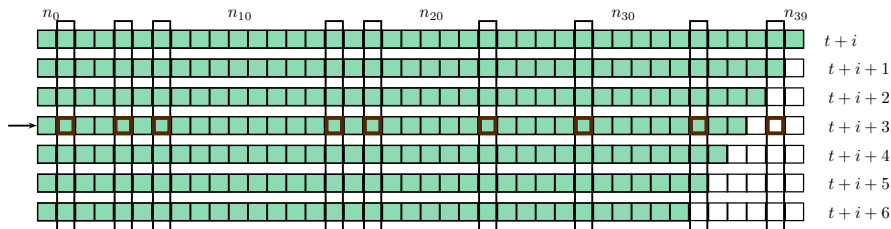


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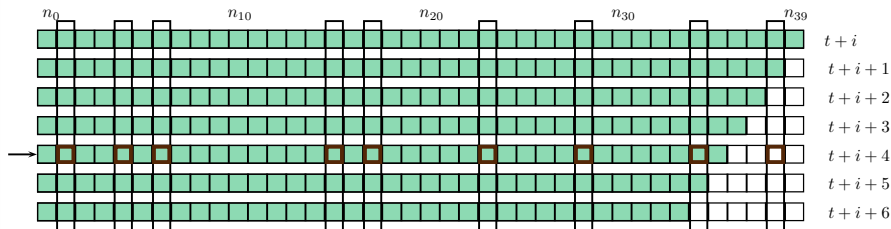


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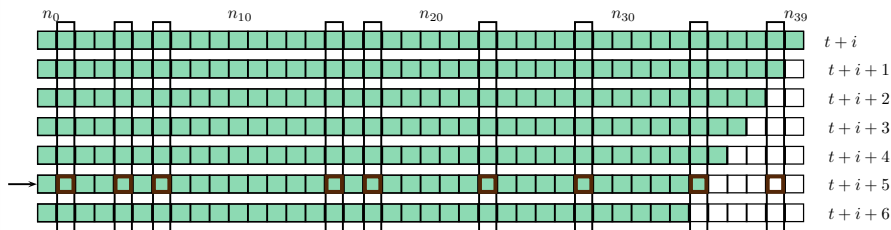


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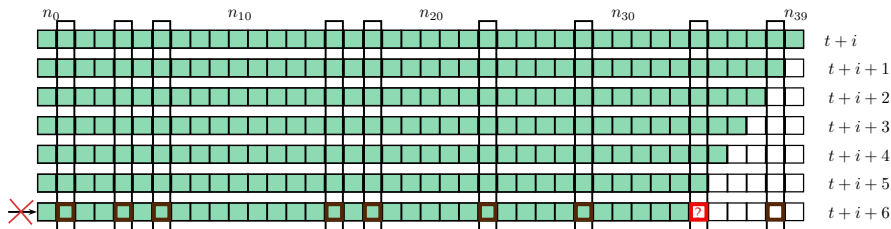


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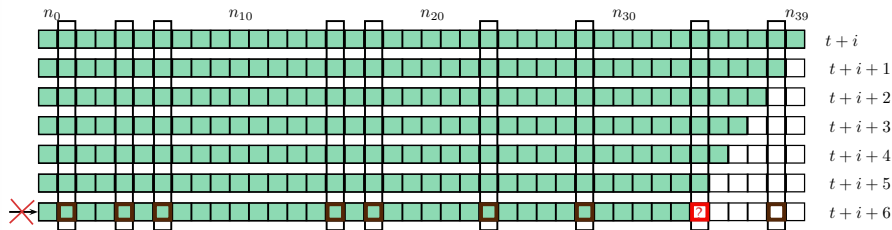


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$n_4^t l_{32}^t = 0$ with probability $3/4$



→ As long as the computation of z_t doesn't imply another unknown state bit

→ 4 rounds

Merging Conditions - Type IV filter

Given an internal state candidate for the NLFSR and for the LFSR at time t

$$z_t = n_4^t l_6^t + l_8^t l_{10}^t + l_{32}^t l_{17}^t + l_{19}^t l_{23}^t + n_4^t n_{38}^t l_{32}^t + l_{30}^t + n_1^t + n_6^t + n_{15}^t + n_{17}^t + n_{23}^t + n_{28}^t + n_{34}^t$$

$n_4^t l_{32}^t = 0$ with probability $3/4$

$n_4^t l_{32}^t = 0$	$n_4^t l_{32}^t = 1$
no need of n_{38}	×
compute z_t	×
compare with the actual one	×
probability of being kept: 1/2	probability of being kept: 1

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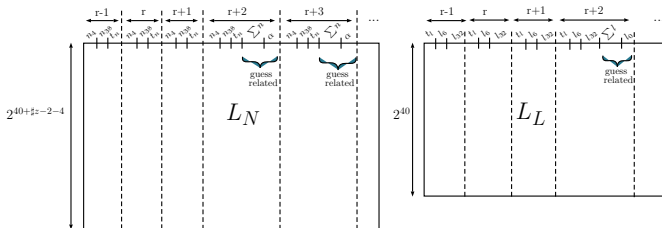
Total probability that a candidate state is kept:

$$3/4 \times \mathbf{1/2} + 1/4 \times \mathbf{1} = 5/8 = 2^{-0.678}$$

→ for 4 rounds total filter of $2^{-2.71}$

Building the lists L_L and L_N

To make this step easier, we **sort** both lists **according to the values of the bits that appear in the sieving relations**



The merge is performed with the **Gradual Matching technique**

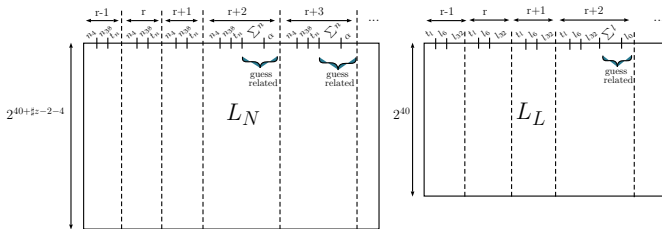


María Naya-Plasencia

How to Improve Rebound Attacks,
CRYPTO 2011.

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→ **We end up with $\simeq 2^{72}$ candidate states with a time equivalent to $\simeq 2^{69}$ encryptions and 13 keystream bits**

Overview

3 main steps:

- 1 Build and arrange two independent lists :
 - the possible internal states for the LFSR at time t
 - the possible internal states for the NLFSR at time t
- 2 Merge them with the help of some keystream bits
- 3 Recover the whole key

Overview

-
-
- 3 Recover the whole key

Key Recovery

Key Recovery

Main Idea: Start from an internal state candidate and go back in time

k_{τ}^* is given by 2 formulas:

$$k_{\tau}^* = n_{39}^{\tau+1} + l_0^{\tau} + c^{\tau} + g(N^{\tau})$$

$$k_{\tau}^* = (k_{\tau} \bmod 80) \times (l_4^{\tau} + l_{21}^{\tau} + l_{37}^{\tau} + n_9^{\tau} + n_{20}^{\tau} + n_{29}^{\tau})$$

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- $k_\tau^* = 1$ and $(l_4^\tau + l_{21}^\tau + l_{37}^\tau + n_9^\tau + n_{20}^\tau + n_{29}^\tau) = 0 \rightarrow$ discard ($p=1/4$)
- $k_\tau^* = 1$ and $(l_4^\tau + l_{21}^\tau + l_{37}^\tau + n_9^\tau + n_{20}^\tau + n_{29}^\tau) = 1 \rightarrow k_\tau \bmod 80 = 1$ ($p=1/4$)
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Repeat until the unique correct key and internal states are found

Note: If this step is done more than 80 times, master key bit involved start repeating

\rightarrow In many cases we can check if the 2 results are consistent

Our Attack: Parameters

- Data complexity = 112 bits
 - 13 bits in the merging part
 - 99 bits in the key recovery part
- Memory complexity: 2^{46} elements of the NLFSR list
- Time complexity $< 2^{70}$

Conclusion

- We exhibit a key recovery attack on Sprout 2^{10} times faster than exhaustive search
- Experimentally verified on a toy version of Sprout
- Open question: Instantiate this design paradigm in a safe way (taking also into account recent independent analyses that showed other weaknesses on Sprout)

Thank you for your attention

